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## SELECTING DESIGN VALUES OF UNDRAINED STRENGTH FOR COHESIONLESS SOILS

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### ABSTRACT

Selecting appropriate design values of undrained strength for cohesionless soils has become an important aspect of many geotechnical evaluations. Transient cyclic or monotonic loads can lead to undrained conditions whereby the resulting strength is considerably reduced from the drained state. Design values of zero strength were commonly selected until about the mid-1980s when both laboratory and field based values were proposed. Today, refinements of the understanding of the undrained phenomenon in cohesionless soils allows an improved and more appropriate means of selecting strengths for these materials.

### RÉSUMÉ

Choisir des valeurs conceptuelles de résistance non drainée pour des sols non cohesifs est devenue une part importante pour de nombreuses évaluations géotechniques. Charges temporaires périodiques ou uniformes peuvent conduire à des conditions non drainées où la résistance résultante est considérablement réduite comparativement à l'état drainé. Des valeurs conceptuelles de résistance nulle étaient généralement choisies jusque vers les années 1980 où les valeurs obtenues au laboratoire et sur le terrain étaient utilisées. Aujourd'hui, une meilleure compréhension du phénomène de non drainage dans des sols non cohesifs offre des moyens améliorés et plus appropriés au concept pour choisir des résistances pour ces matériaux.

## INTRODUCTION

For most loading conditions considered in geotechnical engineering, the load is applied at a slow enough rate to allow drained conditions to exist for cohesionless soils (e.g. sands, sand/silt mixtures and many silts). Undrained shear strength concepts are typically reserved for cohesive deposits where strength characteristics are often described by their undrained strength in relation to effective overburden stress level ( $S_u/\sigma'_{vo}$ ) profile. However, under certain transient loading conditions including seismic events, rapid slope loading or vibratory construction disturbance, undrained conditions may also govern cohesionless soils and estimates of operative strengths are required. Undrained conditions in loose cohesionless soils are commonly termed "liquefaction".

In the early 1970s, undrained strength of loose cohesionless soils, which usually represents the "critical" or "minimum assured" strength condition, was initially thought to be negligible and liquefied soils were attributed zero shear strength under these conditions. For many engineering applications, the assumption of zero shear strength for these materials is unduly conservative and ignores the growing evidence of field and laboratory data in the latter part of the 1970s. The profession began to appreciate that loose cohesionless materials could indeed have undrained strength. Discrepancies between field and laboratory studies in the 1980s (the field *residual* versus laboratory *steady-state* strength issue), however, has led to uncertainty in practice and considerable practical and academic debate about how to address the undrained strength issue.

The latest thought process, developed over the past five years, is to apply a similar strength characterization process to undrained cohesionless soils as is used for cohesive soils. This paper explores the  $S_u/\sigma'_{vo}$  approach for the undrained strength of cohesionless soils and compares the approach with the field performance data and laboratory testing. The methodology removes most of the field-laboratory strength differences of recent debate. However, a simple  $S_u/\sigma'_{vo}$  relationship versus a penetration density measure alone is not enough in itself to allow consistent selection of design strength values. The paper will discuss the reasons for this and suggest methods for arriving at a consistent selection process.

This paper does not attempt to address inertial effects etc. that obviously affect undrained strengths back calculated from flowslides. However, design engineers require confirmation that the body of field and laboratory evidence does indicate that some minimum assured strength exists upon straining liquefied cohesionless materials. This paper summarizes earlier confirmatory work and then presents a more fundamental interpretation of the field and laboratory evidence that provides a rational means of selecting design values of undrained strength.

## HISTORICAL DEVELOPMENT OF UNDRAINED STRENGTH CONCEPTS

Cohesionless soils when saturated can be a major design concern due to the sudden and extreme strength loss these materials can exhibit following a transient load. Liquefaction initiated due to earthquake loading is the most readily recognized situation where this strength loss occurs. Field and laboratory studies over the past 25 years or so have studied the liquefaction phenomenon in detail and shown that both static and dynamic transient loads can trigger liquefaction if certain conditions are present. The main condition, other than saturation, is that the soil exists in a state that will respond to the given load in a brittle fashion.

The methods of assessing whether brittle behaviour will or will not occur for a given deposit or fill all use the state of the material relative to a zero dilatancy state as a discriminator. The state of the material is expressed in many ways depending upon the nature of the data and evaluation procedure and includes void ratio (state parameter), relative density, Standard Penetration Test (SPT) blowcount, Cone Penetration Test (CPT) resistance, and several others. In all cases, there is a common conclusion:

"the further the material state from a state of zero dilatancy (i.e. positive state parameter or loose relative density), the more susceptible the material becomes to brittle behaviour."

Figure 1 demonstrates this concept for the simplified case where the steady state line is linear in void ratio-log stress space. The state parameter,  $\Psi$ , is shown to be positive if the soil is contractant (e.g., moving from  $e$  in-situ to  $e_s$  at steady state). Negative values of  $\Psi$  will develop for initial dilatant soil states upon elevated shear straining.

The literature has produced several terms for the undrained strength of cohesionless soils. Admittedly, some of the terms are specific to different phenomena but this distinction is less important than the manner in which the strength is selected and used in subsequent design. For this paper, the undrained strength will be taken as essentially equivalent to the steady-state strength from laboratory tests or residual strength from field case histories which are both the most common respective terms in the literature. Overall, the strength discussed will be that which is mobilized upon sufficiently straining the deposit in question. The level of shear strain is typically that in excess of  $\gamma = 10\%$ .

The existence of an undrained strength in sandy soils following a transient loading event received little attention until the latter part of the 1970s. Transient loads that caused liquefaction type response such as earthquakes, static loading or unloading, or seepage events, were traditionally designed for by having an adequate limit-equilibrium factor of safety against failure. For the mass that had liquefied, the resisting strength assigned for the analyses varied somewhat but assuming a value of zero (fluid shear strength) and using a full fluid pressure driving force was a common design procedure.

Laboratory research in the late 1960s and early 1970s began reporting values of steady-state strength for sands and cohesionless silts. The most common manner in which to determine this strength was to produce a reconstituted sample and then carry out a series of monotonic, undrained triaxial compressional tests. The stress levels used in these tests were typically very high and, with very few exceptions, in excess of the in-situ stress levels for the design project in question. The steady-state strengths determined were then applied by some assessment of the in-situ void ratio and the relationship developed between this void ratio and the strength.

Field evidence for an undrained strength was mainly developed from shallow flowslides and slumps of embankments and sloping ground where some transient load triggered partial or complete liquefaction. Several groups had assessed the field values empirically (e.g., Lucia et al., 1981) and quasi-theoretically (Jeyapalan et al., 1983) for flowslide analyses. The most commonly cited development, however, was that developed by Seed (1986) and later summarized by Seed and Harder (1990). The development by Seed (1986) related the undrained strength (termed residual strength) to an equivalent clean-sand SPT blowcount at approximately 60% of the theoretical maximum blowcount input energy. The relationship by Seed (1986) and Seed and Harder (1990) proved to provide very low estimates of undrained strength for many situations when compared to laboratory evaluations and field performance data outside their database. Perhaps the two most controversial aspects of the Seed methodology were that:

1. the well-documented San Fernando Dam failures (1971) had quite large discrepancies between the field method and laboratory evaluations; and
2. the methodology suggested a value of zero for the undrained strength below a blowcount of about  $(N_1)_{60-ECS} = 6$  for the lower-bound of the relationship.

In the offices of the first author of this paper, and undoubtedly across the profession, the use of the Seed methodology and the laboratory-field strength dichotomy were issues of large practical concern. In about 1988, in-house re-evaluations of the field and laboratory database started to indicate that the apparent field and laboratory strength dichotomy cited by others did not exist if both sets of data were evaluated in terms of an  $S_u/\sigma'_{vo}$  approach. Although this stress normalization did not remove all of the differences, a large percentage of in-house and published literature fit this approach quite well.

McLeod et al. (1991) published Figure 2 to summarize the original thoughts of the first author of this paper. The numbers on Figure 2 correspond to case histories referenced in McLeod et al. (1991). The range of values from this re-evaluation of case histories yielded:

$$\frac{S_u}{\sigma'_{vo}} = A \cdot (N_1)_{60} \quad (1)$$

where:            A        =        empirical constant  
                    $A_{range}$     =        0.008 to 0.026  
                    $(N_1)_{60}$     =        energy corrected (60% theoretical maximum) SPT at 1 tsf stress level

The practical design significance of the relationship shown in Figure 2 is readily shown in Figure 3. It is seen that the majority of the case histories in the Seed database are at shallow depths of less than 20 metres. For larger cohesionless soil structures that had plausible static or dynamic liquefaction triggers indicated, the choice of design undrained or "residual" strength based upon a database biased towards shallow stress conditions was of great concern. For example, if a tailings impoundment 80 m in height had an  $(N_1)_{60}$  of about 10 blows per 300 mm at that depth, the original Seed (1986) or Seed and Harder (1990) relationship, if adopted, would recommend a design strength of about 18 kPa (~ 350 psf) or even less. The  $S_u/\sigma'_{vo}$  approach would indicate strengths of 175 kPa, roughly an order of magnitude higher.

McLeod et al. (1991), and others such as Jefferies et al. (1990) and Lo et al. (1991), showed that the database itself suggested that confining stress level was a factor in the undrained strength and the  $S_u/\sigma'_{vo}$  approach was an appropriate manner in which to assess the selection of design strengths. More recently, others have followed this line of thinking and the literature has presented several examples using the  $S_u/\sigma'_{vo}$  approach such as Stark and Mesri (1992) and Pillai and Stewart (1993). Figure 4, published by Stark and Mesri (1992), includes the same case histories of McLeod et al (1991) and laboratory data which makes it an interesting review paper.

## STRESS LEVEL DEPENDENCE

The empirical evidence summarized in the previous section pointed to an  $S_u/\sigma'_{vo}$  type of relationship existing for cohesionless soils. However, whether such a relationship based upon soil mechanics observations and principals exists must be addressed. To help address this question, some theoretical aspects and practical observations from field and laboratory evidence are examined below.

The stress level dependence of undrained strength can theoretically be examined using concepts of state in the simplified sense as per Figure 1. If we make the simplifying assumptions that a unique critical state exists for a given soil and that unique state can be described by a straight line in  $e-\log_{10}(p')$  space the examination becomes straightforward. Assuming that the soil starts at the initial state shown with void ratio =  $e$ , movement towards steady-state requires either contraction of the skeleton ( $\Delta e = e - e_{cs}$ ) or a reduction in effective stress due to pore pressure rise. The pore pressure rise occurs from the hydrostatic condition as a process due to the shearing that is occurring under the constraint

of zero volume change (undrained loading). For this loading, the following relation will hold:

$$\Psi = \lambda \log_{10} (p'/p'_{s'}) \quad [2]$$

which, by rearrangement, leads to:

$$p'_{s'} = p' 10^{(-\Psi/\lambda)} \quad [3]$$

where in both Equation 2 and 3  $\Psi$  refers to the state parameter defined by Figure 1 and  $\lambda$  the slope of the simplified steady, or critical, state line in the space shown. Reverting to Mohr type strength definition where  $S$  = the strength of the material, the following is clear:

$$s = p'_{s'} \sin \phi_{s'} \quad [4]$$

or, taking  $\phi_{s'} = 33^\circ$ , for example:

$$s = (0.54) p' 10^{(-\Psi/\lambda)} \quad [5]$$

Therefore, with the simplified example given the strength is linearly related to the initial confining stress,  $p'$ . Moreover, the slope of the linear relationship will be a function of both initial density state ( $\Psi$ ) and material type, gradation and fabric which is expressed by the slope of the steady state line ( $\lambda$ ). The influence of  $\lambda$  in Equation 5 will be discussed in the next section.

From a practical standpoint, the observations available through the existing database as establish in the preceding section of this paper demonstrate that both laboratory and field evidence point to a stress level dependence for the undrained strength of cohesionless soils. Another observation is available through analogy to silts and clays. These finer-grained soils do have cohesion but undrained loading is less a function of cohesion as it is skeletal arrangement and other fabric related strength components.

In cohesive soils, both peak and residual strength profiles show relatively linear undrained strength versus effective confining stress relationships. Without any theoretical appeal, one could surmise that undrained strength behaviour for particulate materials, whether quasi-spherical or platy in shape, should follow some unified construct. Embracing such a concept has not come easily into geotechnical practice. However, evidence from the laboratory and field database which concurs with the theoretical soil mechanics indicates that the concept of stress level dependence for the undrained strength of cohesionless to be favourable.

## CONTROLLING FACTORS OTHER THAN STRESS LEVEL DEPENDENCE

Several of the papers presented earlier have pursued stress level dependence relationships that also use penetration resistance. The latter was the view of the author's until about late 1991. The stress level dependence, as shown in the preceding sections, is both theoretically and practically supported. However, the uses of penetration resistance does not quite capture all of the issue.

Penetration resistance of a given soil is highly influenced by that soils compressibility. As such, strength relationships based solely upon penetration resistance are incomplete. Figure 5 represents a compilation of several natural and man-made (mainly mine tailings) cohesionless soils with fines content (defined as finer than  $74 \mu\text{m}$ ) ranging from 0 to 100%. Note that both the position of the steady-state line, or the relative value of  $\Psi$  for a given hydrostatic state, and the slope of the line,  $\lambda$ , are dependant upon fines content. The largest effect is on position but slope can also change by enough to warrant inclusion in a strength model.

Consequently, any method adopted should ideally make some attempt to include the effect that fabric and gradation have on realized strengths.

## SUGGESTED METHOD FOR SELECTING THE UNDRAINED STRENGTH OF COHESIONLESS SOILS

The suggested method for evaluating undrained strengths for cohesionless soils involves two steps:

- develop an estimate of  $s_v/\sigma'_{v_0}$  from CPT or SPT that includes an assessment of soil gradation; and
- confirm the field estimates with controlled laboratory tests on samples tested at representative densities and confining stress levels.

Figure 6 schematically represents a recent case example using a non-fines corrected penetration resistance versus a normalized undrained strength. The project involved a large ( $\geq 80$  m in height) cycloned tailings impoundment. Both the field and laboratory  $s_v/\sigma'_{v_0}$  values indicated that 0.20 was a prudent design ratio based upon the in-situ state and gradation.

Figure 7 presents a potential relationship for determining undrained strength where density (state) and fines content information is available. This three-dimensional relationship is merely a graphical extension of work such as presented in Figure 6.

The density, as shown on Figure 7, can take the values of  $(N_1)_{60}$ , state parameter ( $\Psi$ ), relative density ( $D_r$ ) or stress normalized CPT tip ( $Q$ ). Fines content can be a literal percentage or an empirical relation to CPT stress normalized friction ratio ( $F$ ). Using

$(N_1)_{60}$  (not fines corrected) and actual measured fines content, cutoff values of  $s_u/\sigma'_{vo} = 0.10$  and  $0.06$  are indicated for most cohesionless soils with the actual valve being gradationally controlled. From  $(N_1)_{60}$  at 6 blows per 300 mm, each line or surface follows a linear extension up to the drained strength which, for most materials, will be at about  $s_u/\sigma'_{vo} = 0.65$  to  $0.70$ . For the 30-60% fines material, which also represents the lower bound strength profile, the following relationship is indicated:

$(N_1)_{60}$ - blows per 300 mm	$s_u/\sigma'_{vo}$
0 - 6	0.06
$6 \approx 30$	$0.06 + 0.0\overset{25}{2}[(N_1)_{60} - 6]$
$\geq 30$	0.65 to 0.70

## SUMMARY

This paper has presented a summary of the state of understanding with respect to the undrained strength of cohesionless soils. Recommended relationships based upon theoretical and practical observations show a stress level dependence for these strengths. In addition, the gradation of the soil can be an important component in the strength assessment.

## ACKNOWLEDGEMENTS

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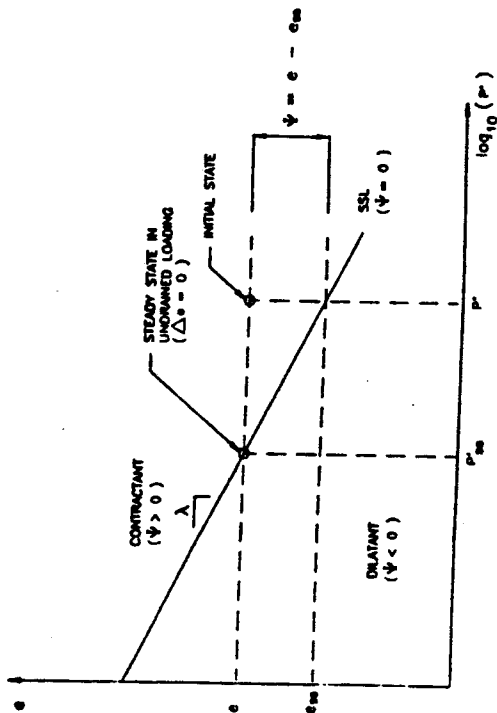


Figure 1. Simplified Soil State Diagram

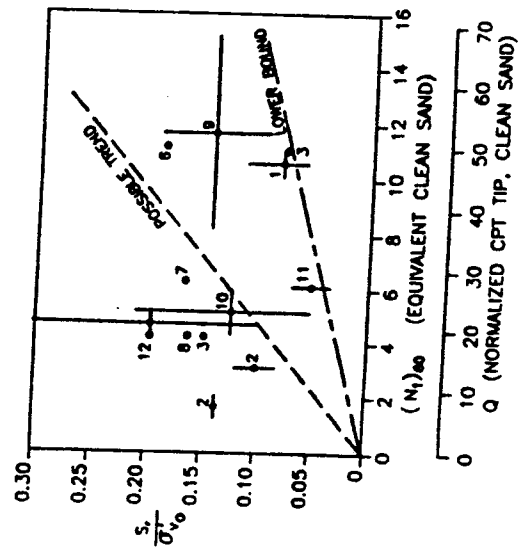


Figure 2. Normalized Undrained Strength from SPT and CPT (after McLeod et al., 1991)

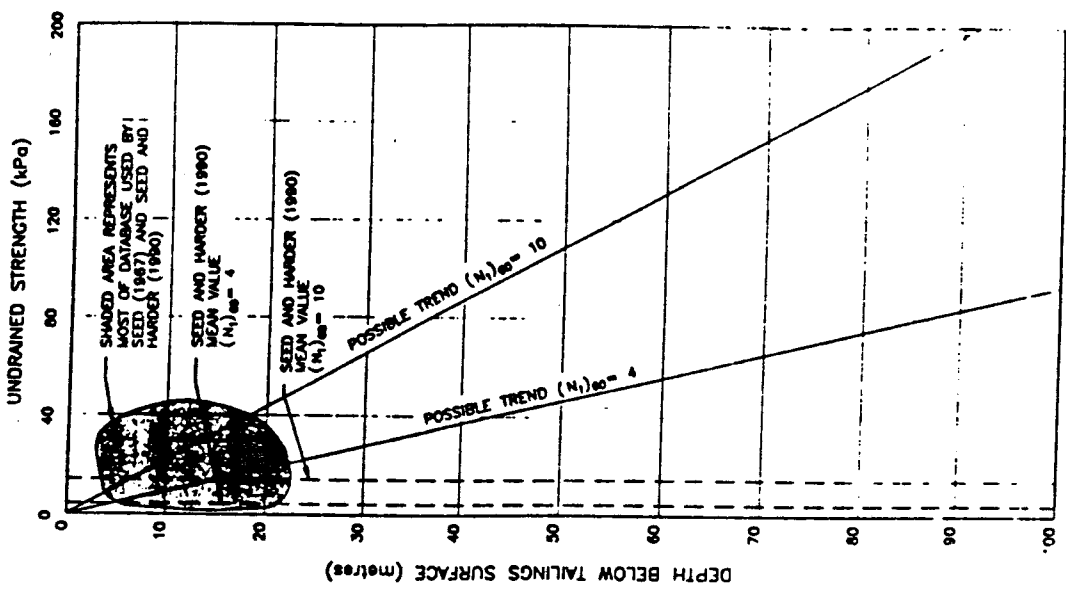


Figure 3. Effect of Stress Normalization on Design Undrained Strength (adapted from McLeod et. al., 1991)

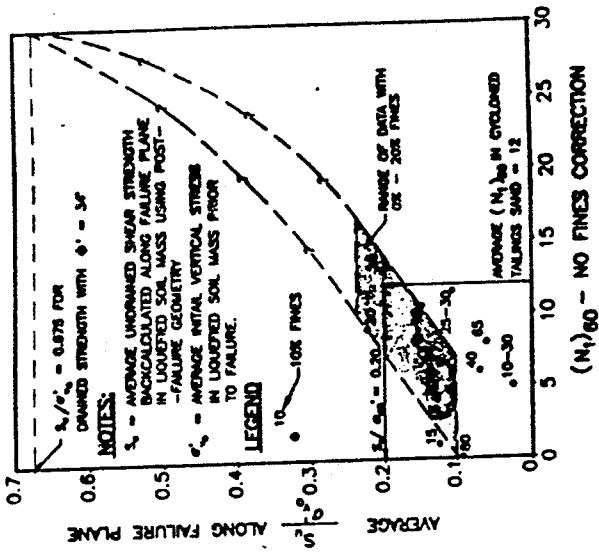


Figure 6. Recent Design Example for Selecting the Undrained Strength of Mine Tailings

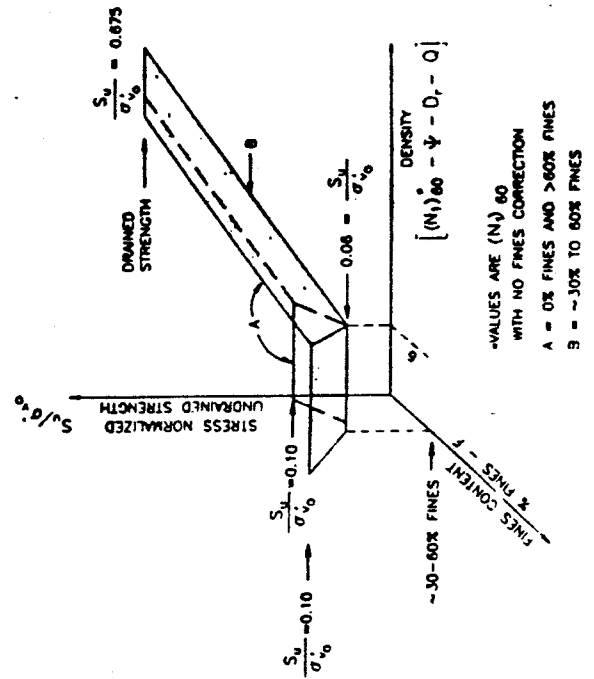


Figure 7. Recommended Relationship for Selecting Design Undrained Strength for Cohesionless Soils

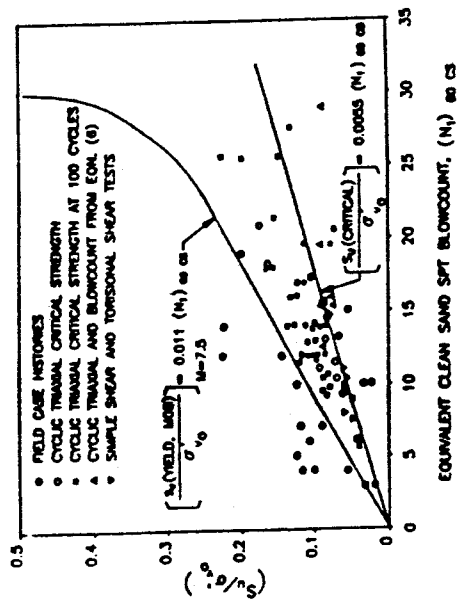


Figure 4. Undrained Strength of Sands - Recent Literature Interpretation (after Stark and Mesri, 1992)

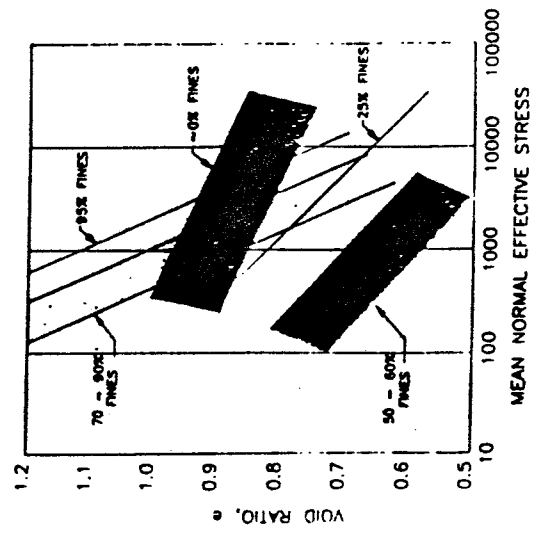


Figure 5. Summary of Laboratory Steady-State Testing Data