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**LOW STRAIN DYNAMIC CHARACTERISTICS OF SOILS WITH THE DOWNHOLE SEISMIC PIEZOCONE PENETROMETER**

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**ABSTRACT:** In-situ soil testing procedures with the downhole seismic piezocone have been under continuous development at the University of British Columbia since 1980. Currently, this rapid and cost effective tool can be used to measure the average dynamic soil properties of shear wave velocity as well as damping with depth along with the interpretation of detailed stratigraphy and geotechnical parameters. The purpose of this paper is to present, in a simple and practical way, the latest seismic piezocone procedures for testing and signal processing to determine low strain damping properties of soil in combination with the usual shear wave velocity measurements. The signal processing for shear wave velocity determination includes the phase velocity technique which gives shear wave velocity as a function of frequency. For the determination of the small strain damping ratio, the spectral ratio slope method is used. Typical results are presented and discussed for a site dominated by peat, organic soils and soft clays, all normally consolidated. Typical results for a sandy site are also presented.

**KEY WORDS:** attenuation, damping, downhole, dynamic site characterization, in-situ measurements, penetrometer, phase velocity, seismic piezocone, shear wave, spectral ratio

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## INTRODUCTION

Obtaining input parameters for geotechnical engineering problems involving the analysis of soils under cyclic loading has traditionally involved laboratory testing or, less rigorously, highly empirical assessments. Notwithstanding the question of field versus laboratory scale, it is very difficult to test soil specimens in the laboratory without significant sample disturbance. Furthermore, it is often not possible to accurately simulate the in-situ stress condition in the laboratory due to both equipment limitations and the lack of a full understanding of the true in-situ stress state. These problems become acute for soils such as very loose sand and peat which are difficult to sample without considerable disturbance. Intensifying the need for in-situ determination of dynamic properties is that these soils which are difficult to sample are also the type of soils which often are of most concern in dynamic site response analyses or in liquefaction studies.

Methods have already been devised for digital signal processing in order to evaluate shear wave velocity,  $V_s$ , from seismic cone penetration test data with pore water pressure measurement (Campanella and Stewart, 1992). Such a test is commonly referred to as an SCPTU (acronym: seismic cone penetration test, with the last letter, U, signifying pore water pressure measurement). This paper also gives a procedure for deriving low strain damping properties from the same data available from the SCPTU provided certain conditions for source and receiver are met. To demonstrate the methodology, an organic-rich very soft soil sequence at a site near Vancouver, British Columbia is evaluated with the procedure.

## ATTENUATION OF SEISMIC WAVES IN SOIL STRATUM

In general, the intensity of a seismic signal in a soil stratum decreases as the distance from the seismic source increases due to wave attenuation. This attenuation is due to geometric spreading and to energy dissipation within the soil mass caused by material damping. Attenuation is usually modelled as viscous damping, where the simulated damping force is assumed to be proportional to the velocity of the soil element. The resulting constant of proportionality is termed the coefficient of viscous damping. The damping ratio,  $D_s$ , is defined as the ratio of the coefficient of viscous damping to a critical value of the coefficient where the motion is attenuated within one wave cycle. The subscript, S, is used in this paper to indicate that the parameter is used for characterizing the behavior of the shear waves. It can be shown that the damping ratio for a material that exhibits a hysteretic behavior in cyclic loading, is the ratio of the area enclosed by the hysteresis loop to  $4\pi$  times the corresponding elastic strain energy. The damping ratio is an important soil property which is required for dynamic analyses when simulating the unload-reload behavior of the soil subjected to a transient loading condition.

## EQUIPMENT AND FIELD PROCEDURES

The piezocone is increasingly being used by practicing engineers as an effective means of geotechnical classification. The method provides a fast, economical and accurate means of

delineating soil stratigraphy and determining geotechnical parameters. The cone has a standard  $10 \text{ cm}^2$ ,  $60^\circ$  conical tip, a  $150 \text{ cm}^2$  friction sleeve and pore pressure transducers which allow transient and dissipation pore pressures to be evaluated at various key locations. Temperature and inclination can also be simultaneously recorded. Initially a seismometer and more recently an accelerometer has been added to the conventional electronic cone penetrometer at the University of British Columbia (UBC) to measure low strain dynamic properties of the soil. Cones with the seismic recording capability, termed seismic piezocones, have also been commercially available for nearly a decade. The initial research at UBC with the seismic piezocone is summarized in Robertson et al. (1986).

The typical arrangement utilized for conducting the SCPTU is shown in Figure 1. As noted above, the modified cone penetrometer with seismic data recording capability, is also capable of providing the static soil properties and stratigraphic information. In contrast, conventional geophysical methods do not provide stratigraphic or *ground profiling* information.

The seismic receiver within the piezocone unit which was used to measure the small strain dynamic properties of soil consists of a piezo-resistive sensor, damped at 70 percent of critical damping (IC Sensors: 3021-002-N accelerometer), which has an essentially flat amplitude response from 0 Hz to about 300 Hz. Mechanical swing hammers are used at UBC as the source of seismic waves although any appropriate and consistent source will work. However, in order to compare the intensity of signals at various depths in a soil profile, a source capable of generating repeatable signals must be used. The repeatability is ensured by selecting a single hammer weight and height of fall for all tests in one sounding. Even with a consistent energy input, particularly in case of soft soil sites, signals contaminated by vibrations from traffic or like sources are occasionally received. In order to eliminate the unwanted noise and maintain a quality signal, blows are repeated at each depth. After removing the spurious data, the rest of the seismic signals obtained from the repetition of blows at a given depth are averaged to increase the signal to noise ratio. For the recording system employed in this study, it was convenient and deemed sufficient to record four signals at each depth for each hammer direction. Additional details of equipments have been given by Stewart and Campanella (1993).

Seismic data are obtained at equally spaced depth intervals of either 0.5 m or 1.0 m. In order to provide for a constant frequency step in a fast Fourier transform (FFT), it is necessary to use the same time step in recording the signals at all depths. Given that the arrival time of seismic waves increases with increasing depth, the time step of recording should be set to enable detection of the shear wave at the greatest anticipated depth of sounding for a given SCPTU. Typically, time steps of 100 to 200  $\mu\text{s}$  have been used. It has also been found to be useful to AC couple the incoming signals to eliminate zero offset. This method essentially performs high-pass filtering at around 2 to 5 Hz.

## SCPTU SIGNAL PROCESSING

For routine shear wave velocity measurements, detailed signal processing is not usually required and the velocities can be easily determined from usually chosen arrival times in most cases. However, for damping measurements, it is necessary to evaluate the quality

and nature of the signal in advance of the calculation. Such signal processing also leads to an understanding of the properties of the measured signals. The signals obtained from the SCPTU can be systematically analyzed in five steps as follows:

#### Step 1: Review of Average

At each depth four sets of seismic signals are obtained by repetitions of the hammer blow. The signal sets obtained at a given depth resemble each other well and the signals which show anomalies with reference to the others are discarded. The seismic signals intended for use in evaluating the small strain damping ratio should be relatively free from interference due to interbedding of soil layers. Stewart (1992) observed that for a stratum characterized by a shear wave velocity between 100 to 200 m/s, a homogeneous soil layer of roughly 3 to 6 m thickness is required for the application of the Spectral Ratio Slope (SRS) method. Therefore, signals showing obvious signs of interference due to the presence of thin layers in the soil profile should only be included in the analysis with caution or eliminated. The remaining signals are then averaged to obtain a data set representative of the given depth.

#### Step 2: Removal of Noise

This step is only necessary at sites where noise contamination of the signals is significant. The main shear wave pulse is examined to identify any obvious noise, which, if present, may be filtered using a rectangular low pass filter. A low pass filter with a flat response up to 100 Hz was often found to be sufficient.

#### Step 3: Selection of the Main Shear Wave Pulse

The nature of the averaged seismic wave signals for the entire sounding depth should normally be reviewed to identify the pulse corresponding to the arrival of shear wave. This exercise warrants caution in the case of very deep downhole (say, greater than 30 m) measurements where detection of the main shear wave pulse becomes difficult because of the large signal attenuation. Surface waves may make detection of the main shear wave pulse difficult at shallow depths less than about 3 m.

#### Step 4: Windowing of the Signal

Once the shear wave arrival for a signal has been identified, a rectangular window is used to isolate the main shear wave pulse. The main shear wave pulse that has been obtained is used in the remaining processing calculations with the tacit assumption that this windowed waveform contains all the characteristics of only the shear wave. The record is padded with zeroes before and after the main shear wave pulse so that the total length of the record in terms of time is kept the same. Windowing is recommended because the smaller pulses that often follow the main shear wave pulse, cannot usually be properly identified (Stewart, 1992).

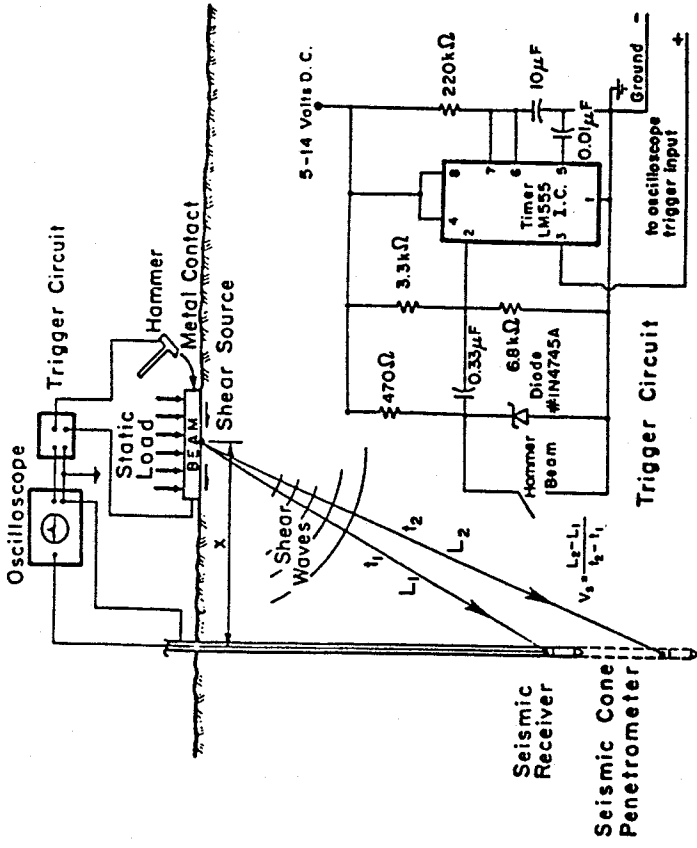


Fig. 1. A Typical Arrangement for Conducting SCPTU (after Stewart and Campanella, 1993)

#### Step 5: Review Fourier Transform for Each Layer

Review of the Fourier transforms of the wave forms are necessary for each distinctive soil layer. Fourier transform of the *clean* shear wave pulse obtained at the end of the preceding step can now be obtained for each depth investigated to identify the frequency range at which the response of all the averaged, windowed and filtered shear wave signals remain approximately similar in shape.

#### CALCULATION OF SHEAR WAVE VELOCITY

As shown schematically in Fig. 1, the shear wave velocity,  $V_s$ , can be calculated by dividing the difference in travel distance between two depths by the time difference between the two recorded signals. The time difference can be found manually by determining the arrival time of the main shear wave pulse or by the *cross-over* technique (Campanella and Stewart, 1992). Alternatively, the time lag can be taken as the time shift of the maximum

cross-correlation of the signals (Campanella and Stewart, 1992). However, the time difference can also be calculated as a function of frequency,  $f$ , using the phase of the cross spectrum of the signals. Dividing this time into the difference in travel distance gives the velocity as a function of frequency,  $V_S(f)$ . The windowed main shear wave pulse as processed in Step 4 above will be used as the input signal for the procedure outlined below.

**Step 1.** The upper (shallow) and the lower (deeper) data sets are transformed to the frequency domain using a fast Fourier transform (FFT) algorithm in the same manner as Step 5 under Signal Processing.

**Step 2.** The complex conjugate of the Fourier transform of the upper data set is then multiplied by the Fourier transform of the lower data set to give the cross spectrum of the signal.

**Step 3.** The phase (in any angular units: degrees or radians) of the cross spectrum can now be evaluated within a range of frequencies of practical importance for geotechnical materials (say, 0 to 250 Hz).

**Step 4.** For each frequency,  $f$ , the time interval can be calculated using the following expression:

$$t(f) = \frac{\text{phase in degrees}}{360^\circ \times f} - \frac{\text{phase in radians}}{2\pi \times f} \quad (1)$$

where

$f$  = frequency, Hz, and

$t(f)$  = time lag at frequency,  $f$ , corresponding to the calculated phase.

**Step 5.** The shear wave velocity at a certain frequency,  $V_S(f)$ , is evaluated by dividing the difference in travel distance of seismic signals,  $d_S$ , by the time lag,  $t(f)$ . The frequency range over which the shear wave velocity remains relatively constant provides the frequency range for which the signals may be considered to be similar, and this velocity can be considered to be appropriate for the depth range between the upper and the lower signals.

#### COMPUTATION OF DAMPING BY THE SPECTRAL RATIO SLOPE (SRS) METHOD

Now that the data have been processed, and phase velocities computed, the procedure for determining the small strain damping ratio is relatively straightforward. The basis of the methodology is the Spectral Ratio Slope (SRS) which has been previously developed and used by others (Redpath et al., 1982, 1986) for interpreting seismic waves.

The damping ratio can be evaluated using the following expressions:

$$k = \frac{A_R}{\partial f \partial d} \left[ -\ln \frac{A_0}{A_0} \right] \quad (2)$$

and

$$D_S = \frac{k V_S}{2\pi} \quad (3)$$

where

$f$  = frequency, Hz,

$d$  = depth, m,

$\partial/(\partial d)$  = slope with respect to depth,

$\partial/(\partial f)$  = slope with respect to frequency,

$A_0, A_R$  = amplitudes of the Fourier transforms of a reference signal and the signal at a depth where the damping ratio is to be evaluated, respectively,

$V_S$  = the shear wave velocity of the layer,

$A_R/A_0$  = Spectral Ratio, and

$D_S$  = the small strain damping ratio of the layer (decimal).

The parameter,  $k$ , in Equation (2) has a unit of  $\text{time} \times \text{length}^{-1}$ . The derivation of Equation (2) and Equation (3) can be found in Stewart (1993) and Redpath et al. (1982, 1986). The essential feature of the double differentiation (first with respect to frequency and then with respect to depth) in Equation (2), is that the radiation damping component is eliminated from the expression. Thus, it is not necessary to assume or attempt to evaluate the amount of radiation damping at each depth. However, a variation of this approach can be used to study the distribution of radiation damping with depth in different soils. The procedure for implementing these expressions to evaluate the small strain damping ratio is given below.

**Step 1.** First, a reference signal is selected. It can be taken as the signal at the shallowest depth within the layer of interest which is not affected by reflection and refraction from the surface.

**Step 2.** Select the *best* frequency range for computation by inspecting the Fourier transforms of all the signals to be analyzed; the range of frequency for which all transforms exhibit a relatively similar response can be selected. Other methods such as the variation of phase velocities with frequency can also assist in the selection. Alternatively, steps 3 through 5 described below may be repeated with different frequency ranges with a goal to obtain the best overall statistics (such as minimization of the standard deviation and/or the difference between the root mean square and mean phase velocities) for the entire depth. The chosen frequency range is kept constant with depth for a particular soil layer.

**Step 3.** At each successive depth, the Fourier transform of the windowed and filtered signal is divided by the Fourier transform of the windowed and filtered reference signal to

evaluate the spectral ratio,  $A_R/A_0$ . The negative of the natural logarithm of this ratio is then calculated and plotted against frequency. The slope of this relationship, approximated as a straight line, is now evaluated within the frequency range selected in Step 2. The computation of  $-\ln(A_R/A_0)$  is repeated for the same frequency range for all of the seismic signals from the soil layer of interest. Different frequency ranges can be used for different soil layers, which may be necessary where there is a large difference in stiffness between adjacent soil layers (e.g. peat underlain by dense sand).

Step 4. The slopes of the spectral ratios with respect to frequency (which are taken to approximate the first partial derivative of the spectral ratio with respect to frequency) are then plotted against depth. The slope of this plot can then be evaluated. This slope is taken as an approximation of the second order partial derivative of  $-\ln(A_R/A_0)$  with respect to frequency and depth (Equation 2).

Step 5. The damping ratio for the layer can now be evaluated using Equation (3) from the knowledge of the phase velocity of shear waves evaluated in Step 5 of the preceding section. As will be shown below, it is often necessary to divide the site into two or more layers due to stiffness variations as noted in Step 3. A detailed worked example of this procedure is given in Stewart and Campanella, 1993.

#### ILLUSTRATION OF THE PROCEDURE

The procedure described above was used to compute the low strain damping ratio for a site consisting primarily of organic soils at Burnaby, British Columbia. The Burnaby valley, where the site is located, is an erosional valley formed on a highly overconsolidated silty to sandy glacial till. Post-glacial deposits of marine clays and organic swamp deposits (Armstrong, 1956) lay over the till. The fine grained fluvial sediments are possibly associated with the retreat of the glaciers. The sensitivity of this deposit owes its origin to deposition in a saline environment, and subsequent isostatic rebound and leaching.

The site at which the SCPTU investigation was carried out lies at the intersection of the Trans Canada Highway and Kensington Avenue. A detailed stratigraphic profile of the site as obtained from the SCPTU is shown on Fig. 2. The site can be broadly characterized by two layers to roughly 18 m depth. Beneath a surficial fill there is an organic-rich material. This organic layer is predominantly peat to a depth of about 5 m, becoming an organic-rich silt to silty clay as the mineral content of the layer increases. The organics also show a fibrous to amorphous trend with depth. This organic rich layer is underlain by a layer of very soft sensitive clay to silty clay which is uniform as shown by the constant friction ratio and linear penetration pore water pressure response. The undrained shear strength,  $s_u$ , for the top layer of organic soil is lower than 20 kPa while that for the bottom clay layer is lower than 15 kPa. The method for evaluation of the low strain damping ratio described earlier was automated by implementing it into macros in the digital signal processing program VU-POINT II (Maxwell Laboratories, Inc., 1991). These automated procedures were then utilized to evaluate the shear wave characteristics and small strain damping ratio for the soils between depths 2.5 and 15.5 m. Fig. 3 illustrates two seismic signals from the data set. The windowed signals were then low pass filtered using a filter characterized by a very flat

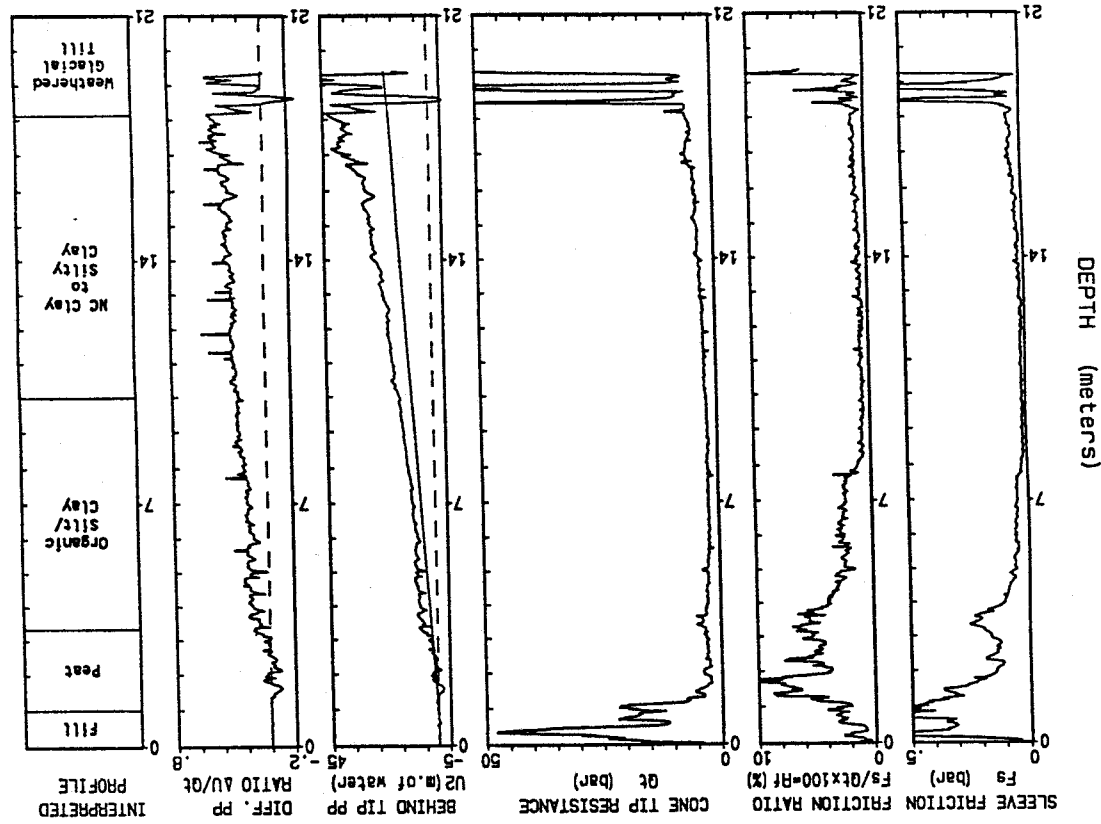


Fig. 2. Piezocene Data from the SCPTU (1 bar = 100 kPa = 0.1 MPa  $\approx$  1ts)

response between 0 and 90 Hz and a steep decay in the region between 90 and 110 Hz (Fig. 4).

A frequency range between 5 and 35 Hz was used for the calculation of average shear wave velocities. The choice of this range was made after examining the apparent relationship between the shear wave velocity,  $V_s$ , and frequency,  $f$  (Fig. 5). The results of the phase velocity computation for the main shear wave pulse are shown in Fig. 6. The univariate statistics shown pertain to the variation of the phase velocity against frequency within the selected frequency range. The shear wave velocity in the peat is about 25 m/s and that in soft clay varies between 20 m/s at 7 m depth and 60 m/s at 15 m depth. The low values of shear wave velocity are typical of weak soils like the ones encountered at this site.

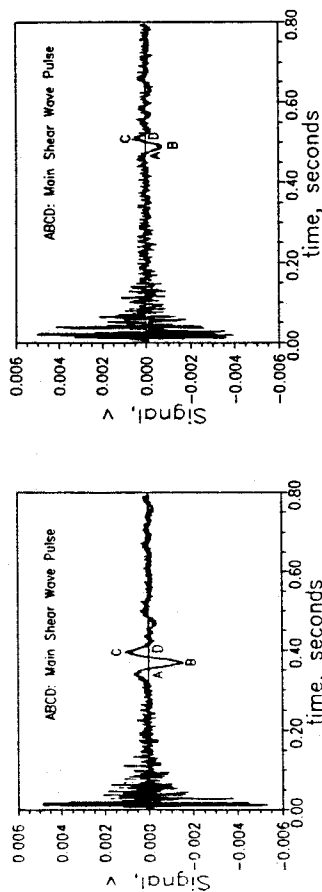
For the computation of the small strain damping ratio for the organic layers at shallower depths and the lower clay layer, the seismic signals at 1.9 m and 6.4 m depth were chosen as reference. The frequency range used here was the same as in the preceding step (5 to 35 Hz). As noted earlier, the soil profile of the site consists of two major layers, between depths 2.4 and 4.4 m, and between 10.0 and 15.0 m. A transition layer between 4.4 and 10.0 m is also apparent. Consequently, an attempt was made to fit the  $-\ln(A_p/A_0)$  versus depth data from these three soil units using three linear relationships (Fig. 6). The slopes with respect to depth of the linear regression fits AB, CD and EF are 0.00588, 0.01216 and 0.00159, respectively; the values of  $k$  (Equation 2) for these three layers in s/m units. It may be noted that a considerable scatter exists in  $-\ln(A_p/A_0)$  versus depth data at interlayer boundaries due to surface disturbance such as scattering, reflection and refraction.

The shear wave velocity and the appropriate value of  $k$  of the layer can now be substituted into Equation (3) to calculate the small strain damping ratio. Using average values of shear wave velocity of 25.0 and 46.3 m/s for data from depths above and below 10.0 m, the small strain damping ratios for the three layers of computation are:

- 2.4 m to 4.4 m depth: 2 percent,
- 8.4 m to 9.9 m depth: 5 percent and
- below 10.0 m depth: 1 percent, respectively.

The computed small strain damping ratios are within the range of reported values for cohesive soils (Stewart and Campanella, 1993). The results might have been affected by scattering for the transitional layer between depths 8.4 and 9.9 m.

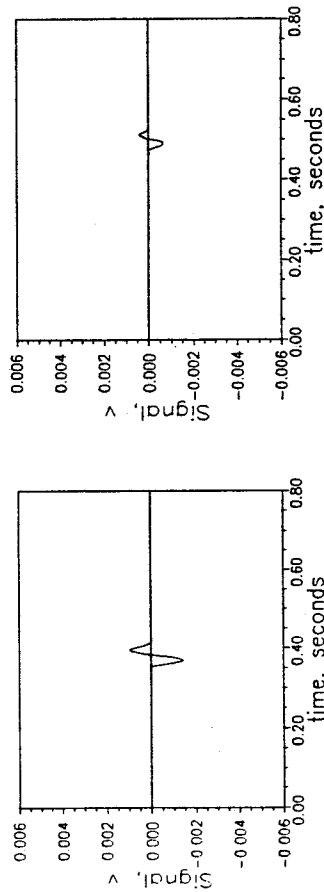
The SRS method was also applied to the SPTU data from McDonald Farm, a site near Vancouver International Airport in British Columbia, for estimating the dynamic soil properties (Campanella and Stewart, 1993). The stratigraphy at this location consists of a uniform sand layer below a 1 m thick surficial fill and a 3 m thick overconsolidated crust of silty sand. The depth of water table is about 1 m. The sand layer extends up to a depth of 15 m, below which there is a transitional layer of sand to clayey silt. A uniform clayey silt layer exists below 18 m depth. The sand layer is mainly fine-grained with a mean grain size of about 0.2 mm. The average relative density of this layer is 50 percent. Based on the cone bearing data, the estimated undrained shear strength,  $s_u$ , for the clayey silt layer varies between 50 and 75 kPa. Fig. 8 summarizes the computed small strain damping properties of the sand and the clayey silt layers. Typically, seismic data from a site which consists of inorganic soil layers (e.g., McDonald Farm) is easier to interpret and analyze.



(a) at 9.9 m depth

(b) at 14.4 m depth

Fig. 3. Averaged Time History of Seismic Signals at Two Depths



(a) at 9.9 m depth

(b) at 14.4 m depth

Fig. 4. Clean and Windowed Shear Wave Pulse for the Data Given In Fig. 3.

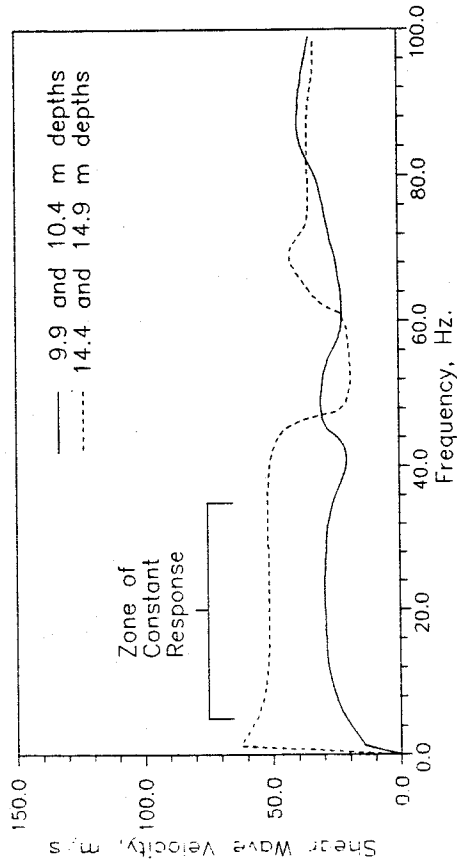


Fig. 5. Shear Wave Velocities as Functions of Frequencies

## CONCLUSIONS

The use of the seismic piezocone (SCPTU) to measure stratigraphic details and shear wave velocity is well established. The addition of a repeatable source, a flat response receiver and appropriate signal processing also allow the determination of low strain damping ratio from measured signals. Signal averaging, filtering and windowing of the main shear wave pulse improve the information used in the procedure. Calculation of the shear wave velocity using the phase of the cross spectrum shows the variation of velocity with frequency, and provides information on the usable frequency range. The spectral ratio slope (SRS) method described in this paper can be used to calculate average low strain material damping from downhole seismic data.

The SRS method was employed to measure the small strain dynamic characteristics of organic soft soils at a location near a busy roadway. The traffic generated noise was removed by repeated comparison of signals, selective rejection, averaging of four signals and digital filtering. The average shear wave velocities measured were 25.0 and 46.3 m/s, respectively, for layers above and below 10.0 m depth. There was considerable scatter, probably due to soil layering, in the damping results obtained from depths between 4.9 and 8.4 m. In the relatively uniform clay there was little scatter and the calculated value of small strain damping ratio was 1.0 percent. Small strain damping ratios were also computed for layers from depths of 2.4 to 4.4 m, and 8.4 to 9.9 m. These layers have significant organic content and are stratified.

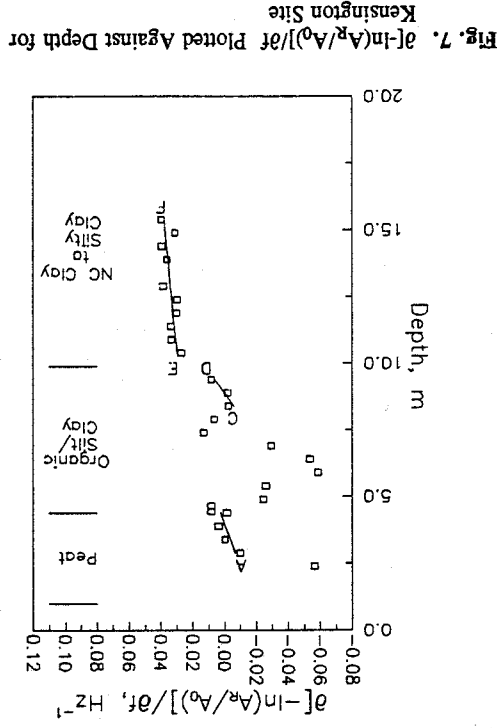
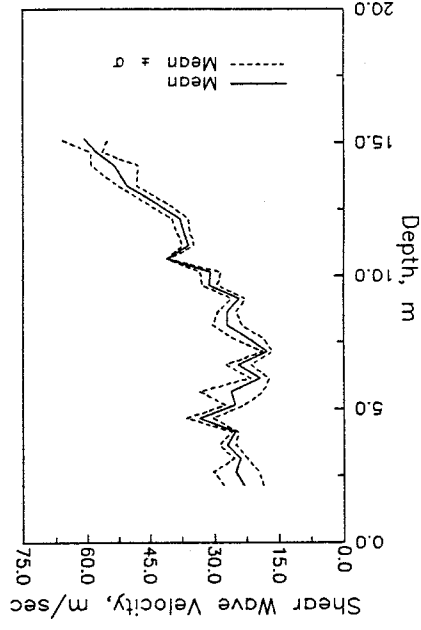


Fig. 7.  $[-\ln(A_r/A_0)]/\delta t$  Plotted Against Depth for Kensington Site

Fig. 6. Shear Wave Velocities at the Kensington Site



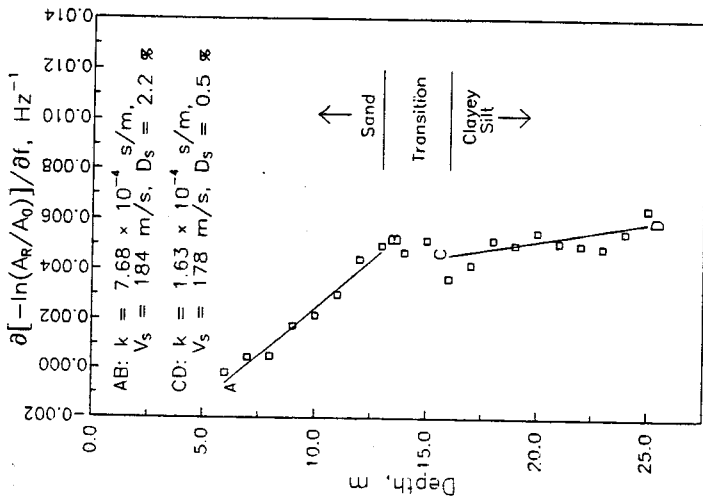


Fig. 8. Small Strain Damping Ratio from SRS Profiles: Vancouver International Airport Site (after Stewart, 1992)

The calculated damping ratios for these layers were 2.0 and 5.0 percent, respectively. These measurements were likely to have been affected by layering.

The small strain damping ratios were also presented for a more typical soil profile at McDonald Farm consisting of a uniform sand layer underlain by a transitional layer followed by a uniform layer of clayey silt extending to a depth of 25 m. The damping results from McDonald Farm had much less scatter and were much easier to analyze compared to the Kensington site. The values of small strain dynamic properties computed for both sites agree well with published reports on similar soils.

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