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Scale of fluctuation as a descriptor of soil variability

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ABSTRACT: Variability of soil properties is traditionally expressed by the mean and standard deviation of a soil stratum. The importance of the scale of fluctuation in the evaluation of spatial variability of soil profiles is highlighted in the paper which also suggests a modified and more convenient method of determining the scale of fluctuation. The scale of fluctuation has been applied to cone bearing, sleeve friction and pore pressure measurements obtained from the cone penetration test to determine the averaging effects of the above properties. The scale of fluctuation of soil properties is also compared with the conventional coefficient of variation and the advantages of the former method are discussed.

1 INTRODUCTION

In geotechnical engineering it is common to divide the heterogeneous soil stratum into statistically homogeneous layers. The means of these statistically homogeneous layers are then considered for design and analysis, completely neglecting the effect of variation or fluctuation about these values. A constant mean, constant standard deviation and an autocorrelation function which is independent of the location and dependent only on the separation distance (lag distance) are necessary requirements for homogeneity or stationarity. If a soil stratum exhibits varying types of trends at different depths, it can be divided into distinct layers, each identified by a particular trend; linear, curvilinear,etc. A trend in actual effect is a non constant mean and therefore in keeping with the above mentioned definition of stationarily, it will have to be removed, to render the soil layer to be stationary. If the trend is not significant, standardization alone may suffice, but otherwise trend analysis will have to be performed after identifying the different layers in the stratum. These layers are then treated individually, in order to derive their respective statistics.

In addition to the mean Q_m, two other parameters are required to describe the spatial variability of a soil property characteristic which is to be treated as random (Vanmarke - 1977). One of these parameters is the standard deviation (σ) which measures the degree to which the actual values differ from the mean. The coefficient of variation (η) is a standardized form of a variability factor, which gives the relationship betweeen the mean and the standard deviation. η is defined as,

$$\eta = \frac{\sigma}{Q_m} \tag{1}$$

The scale of fluctuation, δ, is the other important parameter. This measures the distance within which the soil properties show strong correlation. If two points in a soil layer lie closer than its scale of fluctuation, the soil property values at these two points will be either higher or lower than the mean of the layer. It is in this sense that δ is also known as the distance of perfect correlation. A low value of the scale of fluctuation means rapid fluctuations of the property value about the mean (high variability) and a high value of δ reflects the slowly varying nature of the property value about the mean (low variability). The above explanation about the scale of fluctuation and its significance, explains why it is important to consider it, in addition to the mean and the standard deviation, when a soil profile needs to be fully characterized.

In addition to using the scale of fluctuation merely to represent soil variability, it has found several important applications when soil properties are treated as random fields. These include the correlation between spatial averages (Vanmarke -1983), and exceedance probabilities of single state (Vanmarke - 1978, '83) and multi-state (Tang - 1983) media.

The concept of the scale of fluctuation has been applied to a series of cone penetration tests (CPT), obtained in and around Vancouver, Canada. The data have been acquired at 2.5 cm intervals, using a cone with a base area of 10 sq. cm, penetrating at 2 cm/sec. More details of the CPT is given in Campanella et al. (1983).

2 VARIANCE FUNCTION (Γ^2.(Z))

The scale of fluctuation is derived in terms of the variance function which adequately explains the effects of spatial averaging. The procedure as described Vanmarke (1983) is as follows;

The data is first considered in pairs (n=2) and a moving average series for the data is obtained where the length of averaging will be equal to the spacing of data points, (Z_2). The standard deviation, (σ_2) of this series is also calculated. σ_2 will be lower than the standard deviation of the original data set, σ, due to the cancelling out of fluctuations due to spatial averaging. The above procedure is extended to the case n=3, and the corresponding standard deviation of the series, σ_3, is calculated with the spacing, Z_3, being equal to twice the spacing of the original data points. For a typical CPT sounding which samples at 2.5 cm, Z_2 will be equal to 2.5 cm and Z_3 will be 5.0 cm. This procedure is repeated for n=3, 4, 5, ... until n approaches the total number of data, N. The effect of spatial averaging will be more significant with increasing n with $\sigma > \sigma_2 > \sigma_3 > \ldots.\sigma_n$.

For each n, the variance function, can be calculated from,

$$\Gamma^2 (Z_n) = \frac{\sigma_n^2}{\sigma^2} \qquad (2)$$

where, σ_n^2 is the variance (squared of the standard deviation) of the derived moving average series of degree n, and σ^2 is the variance of the original data. If the spacing of the data is d, Z_n in Eq. 2 will be equal to (n-1)d. The variance function given above can be determined for different lag distances (separation distances), Z. Figure 1 illustrates a typical variation of Γ^2(Z) which has a maximum value of unity, decaying towards zero for increasing lag distance values Z. Vanmarke (1978) has explained in detail, how the scale of fluctuation, δ, can be obtained.

For large values of Z (very large n),

$$\Gamma^2 (Z) . Z = \delta \qquad (3)$$

In the method recommended, the value of Γ^2(Z) is picked from the curve (Fig. 1), at a reasonably high value of Z, where there is a distinct change in the curve. The point of inflexion is a good choice. The variance function in Fig. 1 has a value of 0.27 at the point of inflexion where Z = 1.2. From Eq.3 , the scale of fluctuation was determined as 32.40 (1.2m x 0.27) cm.

A practical variant of the above method of determining δ is used in this paper (Wickremesinghe - 1989). It makes use of Eq. 3 directly and is very convenient for computer applications. At large values of Z, the function Γ^2(Z).Z reaches a peak and this maximum value gives a very good approximation for δ. Figure 2, shows the variation of Γ^2(Z).Z with Z and as can be observed reaches a maximum value of 31.25 cm, which compares very well with the value obtained from the method recommended by Vanmarke (1977). The above values of δ, relate to the cone bearing. Values of scale of fluctuation obtained for friction and pore pressure for different data sets are tabulated in Table 1. These values of δ are for standardised data.

The layer depths chosen are indicated in parenthesis in the table which also gives a comparison of the method, for the CPT bearing profile given in Vanmarke (1977). All the results in Table 1 indicate the adequacy of the method suggested. Figures 3, 4 and 5 illustrate the CPT profiles for the Haney, Langley and Strong Pit sites for which the δ values have been calculated and tabulated in Table 1.

3 REMOVAL OF TREND

Soil properties are highly depth dependent and hence CPT parameters such as cone bearing, sleeve friction and pore pressure exhibit significant trends with depth. If the trends are not significant, data can be stationarized by standardising. In the presence of significant trends, these will have to be removed, prior to determining the scale of fluctuation. A pre-requisite to this of course is the identification of statistically distinct layers.

Bearing and friction exhibit linear trends while it is usual for the pore pressure profile to possess a curvilinear trend (Figs. 3, 4 and 5). If a linear trend is used to stationarize a pore pressure profile, the resulting residuals will not be stationary, due to the curvilinear effects of the trend not being removed. This will give rise to an incorrect value for δ. The decision as to what type of trend removal is necessary could be taken by inspecting the residuals. If a linear trend removal results in a stationary set of residuals, curvilinear trend analysis is unnecessary.

The bearing and friction profiles generally show a linear trend as can be observed from Figs.3 to 5. In certain profiles where the curvature of the pore pressure profile is prominent, the scale of fluctuation obtained for the linear trend removed data and the curvilinear trend removal data show an appreciable difference (Table 2), with the latter method giving lower values.

As already explained, the curvilinear trend would always be more suitable, since it also includes the linear case as a subset and Table 2 provides sufficient evidence, as to why a curvilinear trend removal method has to be adopted, if the trend cannot be adequately represented by a straight line. The δ for the curvilinear trend is less with the only exception being for the result of Langley 3 data which gives a slightly higher value of δ. The reason for this anomaly is that in the case of the Langley 3 data the pore pressure is better represented by a linear trend, than by a curvilinear trend. The Langley 2 data seem to have very close δ values for the two methods of trend removal, suggesting that the pore pressure profile may be adequately represented by either a linear or a curvilinear trend.

Figure 6 illustrates the relationship of the variance function times lag distance ($\Gamma^2(Z).Z$) for the Strong Pit data for bearing, friction and pore pressure. The maxima of the respective curves give the values of the scale of fluctuation (Table 3).

4 AVERAGING EFFECTS OF BEARING, FRICTION AND PORE PRESSURE

The sleeve friction value obtained from the CPT is an average value of the sleeve friction extending across the length of the friction sleeve. Due to this averaging effect, the scale of fluctuation could be expected to be higher than if it was measuring values at a distinct point. This is because the averaging process would cancel out fluctuations, resulting in a lower variability. The cone bearing, however, is theoretically expected to give a lower scale of fluctation (higher variability) since cone bearing is measured at a singular point at the cone tip.

Results in Table 3 agree with the above explanation, because the δ values for bearing is less than the corresponding values for friction. It is also interesting to note that the δ values for bearing are not as low as those for pore pressure. The pore pressure, measures values over the length of the sensing element, which is about 5 mm and for all practical purposes can be considered as measuring values at a single point. If the cone bearing too was measuring values at a single point, the δ value relating to it would also be as low as that of pore pressure. It is evident from Table 3 that it is not, suggesting that the bearing from the CPT too is indicative of a value which is averaged out over some length. However, the fact that the bearing δ value is not as high as that for friction also suggests that the bearing value from the CPT is representative of a value avearged over some length which is less than the averaging length for friction. The above argument that the cone bearing value too is more indicative of the bearing over a finite length, instead of the value at a point, is supported by the fact that it is widely acknowledged (Campanella et al. - 1983) that the cone bearing at a point is dependent not only on the soil property at the cone tip, but also on immediately past and future values.

The value of δ for pore pressure is less than that for friction and bearing, indicating the variable nature of this parameter. In all the cases without exception, results tabulated in Table 3 show that δ for friction is greater than that for bearing which is greater than the δ value for pore pressure.

5 SCALE OF FLUCTUATION AND VARIABILITY

The scale of fluctuation adequately represents the variability of a layer. The variation of $\Gamma^2(Z).Z$ with Z, which gives the value of δ for these three layers are illustrated in Fig 7. Higher the variability of a layer, more fluctuations about the mean are expected, resulting in a relatively low value of δ. On the other hand a slowly varying fluctuating component about the mean, represents low variability, giving rise to a relatively high value of δ. Figure 9 illustrates the coefficient of variation profile for the bearing profile (Figure 8) which has been divided into three layers. The average values of the coefficient of variation and the δ values of the three layers are tabulated in Table 4 which indicates the relationship between δ and the coefficient of variation.

Layer 2 which has the lowest variabilty ($n=0.078$) has the highest δ value of 41.75 cm. Layer 3 has the second highest variabilty ($n=0.180$) and also the second highest value of δ (= 32.63 cm). Layer 1 is the most variable ($n=0.190$) and appropriately has the lowest δ value of 29.37 cm. The variabilities of Layers 1 and 3 are however, not very different as also reflected in their values of scale fluctuation, providing further evidence of the close relationship between the coefficient of variation and the scale of fluctuation.

6 CONCLUSIONS

The following conclusions could be arrived from the discussions in this paper.

(1) The suggested method of determining the scale of fluctuation compares well with the method suggested by Vanmarke (1977), and is more adaptable to be used in computer applications.

(2) The scale of fluctuation obtained from cone bearing is lower than that of sleeve friction,

Table 1. Comparison of the Two Methods for Obtaining the Scale of Fluctuation for Data Subject to Standardisation.

Data	Parameter	Vanmarke's Method	Present Study
Haney 2 (9.3 - 15.50 m)	Cone Bearing Sleeve Friction Pore Pressure	32.34 33.60 30.45	31.25 35.99 26.68
Langley 3 (2.60 - 10.80 m)	Cone Bearing Sleeve Friction Pore Pressure	23.04 41.79 17.91	24.08 39.98 21.98
Strong Pit (5.25 - 10.12 m)	Cone Bearing Sleeve Friction Pore Pressure	27.99 37.59 13.76	26.22 36.88 17.75
Vanmarke (1977)	Cone Bearing	120.0	92.2

Table 2. Comparison of the Scale of Fluctuation for Pore Pressure Using Linear and Curvilinear Trend Removal.

Data	Layer Depths (m)	Linear Trend	Curvilinear Trend
Haney 1	7.15 - 13.12	34.85	27.21
Haney 2	9.30 - 15.50	34.21	30.45
Haney 3	13.52 - 22.50	35.34	28.45
Langley 1	2.60 - 10.75	46.08	26.15
Langley 2	2.60 - 10.60	37.37	37.33
Langley 3	2.60 - 10.80	17.91	18.41
Strong Pit	5.25 - 10.12	20.18	13.76

Table 3. Comparison of the Scale of Fluctuation for Cone Bearing, Sleeve Friction and Pore Pressure.

Data	Layer Depths (m)	Cone Bearing	Sleeve Friction	Pore Pressure
Haney 1	7.15 - 13.12	31.60	38.00	27.21
Haney 2	9.30 - 15.50	31.25	35.99	30.45
Haney 3	13.52 - 22.50	29.06	35.34	28.45
Langley 3	2.60 - 10.80	24.08	39.98	17.91
Strong Pit	5.25 - 10.12	26.22	36.88	13.76

Table 4. Relationship of the Scale of Fluctuation and the Coefficient of Variation.

Layer	Layer Depths (m)	Scale of Fluctuation, δ (cm)	Coefficient of Variation v
1	1.17-4.50	29.37	0.190
2	4.50-10.12	41.75	0.078
3	10.12-13.50	32.63	0.180

indicating that the averaging effects are lower for bearing than friction. However, the scale of fluctuation for pore pressure is lower than that obtained for bearing, suggesting that the value obtained for cone bearing from the CPT, is not indicative of a value measured at a single point, unlike pore pressure.

(3) The scale of fluctuation of a soil layer is a very efficient descriptor of variability and is inversely proportional to the coefficient of variation.

REFERENCES

Campanella RG, Robertson, PK and Gillespie, DJ (1983) - Cone Penetration Testing in Deltaic Soils , Canadian Geotechnical Journal, Vol. 20, No. 1, pp. 23 - 35.

Tang WH and Sidi I (1984) - Random Field Model of A Two State Medium, Proceedings of the Fourth ASCE Specialty Conference on Probabilistic Mechanics and Structural Reliability, Berkely, CA, USA, Ed. Y K. Wen, pp. 210 - 214.

Vanmarke EH (1977) - Probabilistic Modeling of Soil Profiles, Journal of the Geotechnical Engineering Div., ASCE , Vol. 103, No. GT 11, pp. 1227 - 1246.

Vanmarke EH (1978) - Probabilistic Characterization of Soil Profiles, Proceedings of the ASCE Specialty Conference on Site Characterization and Exploration, Northwestern University, Illinois, USA, pp. 199 - 219.

Vanmarke EH (1983) - Random Fields : Analysis and Synthesis, MIT Press, Massachussettes, USA.

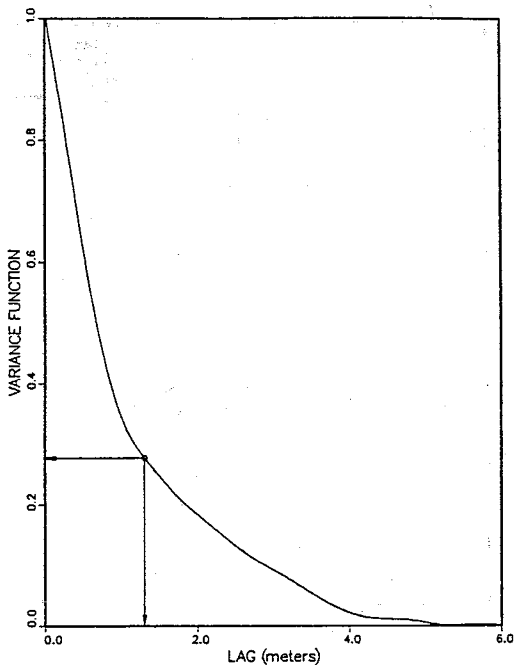

Figure 1: Variance Function of Haney Data.

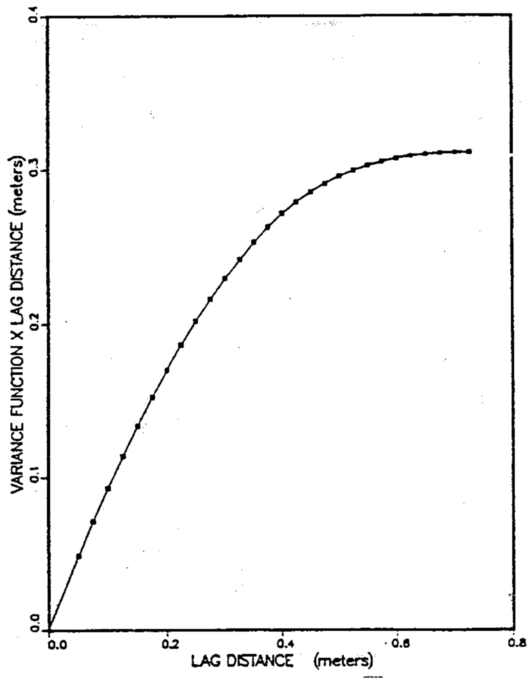

Figure 2: Variation of the Variance Function times Lag Distance for Haney Data.

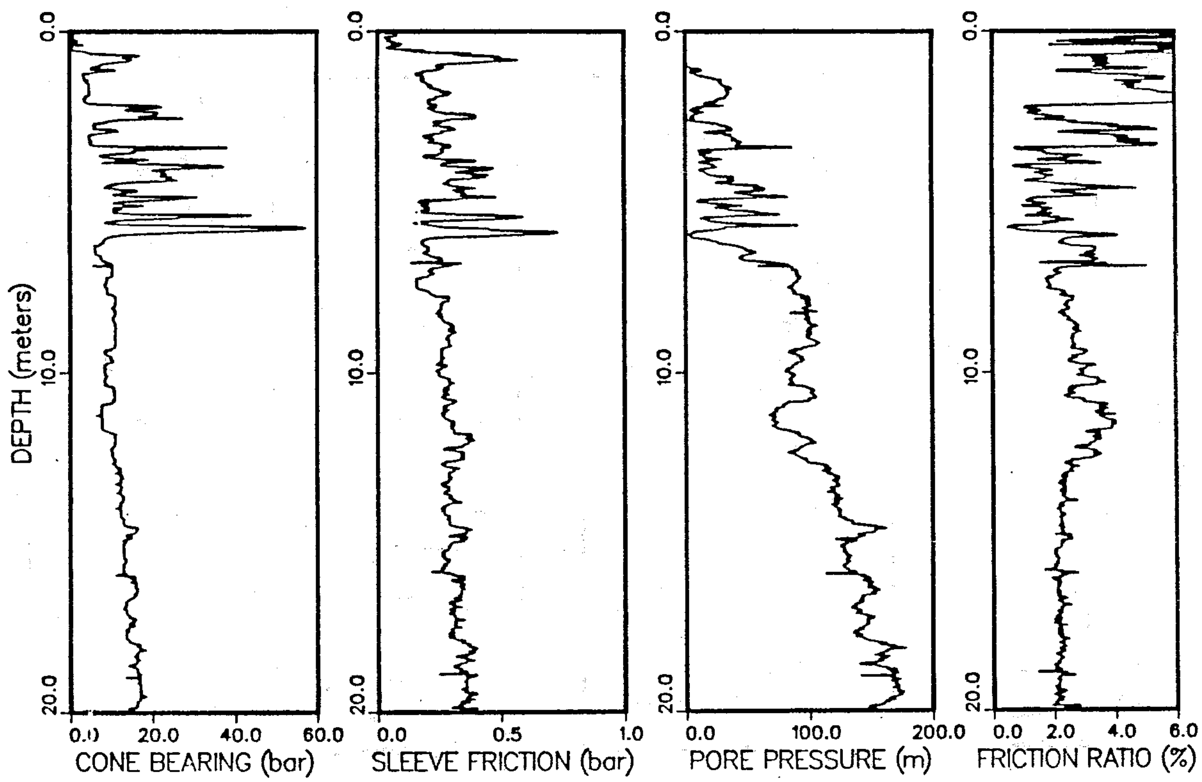

Figure 3: Cone Bearing, Sleeve Friction, Pore Pressure and Friction Ratio Profiles at Haney Site.

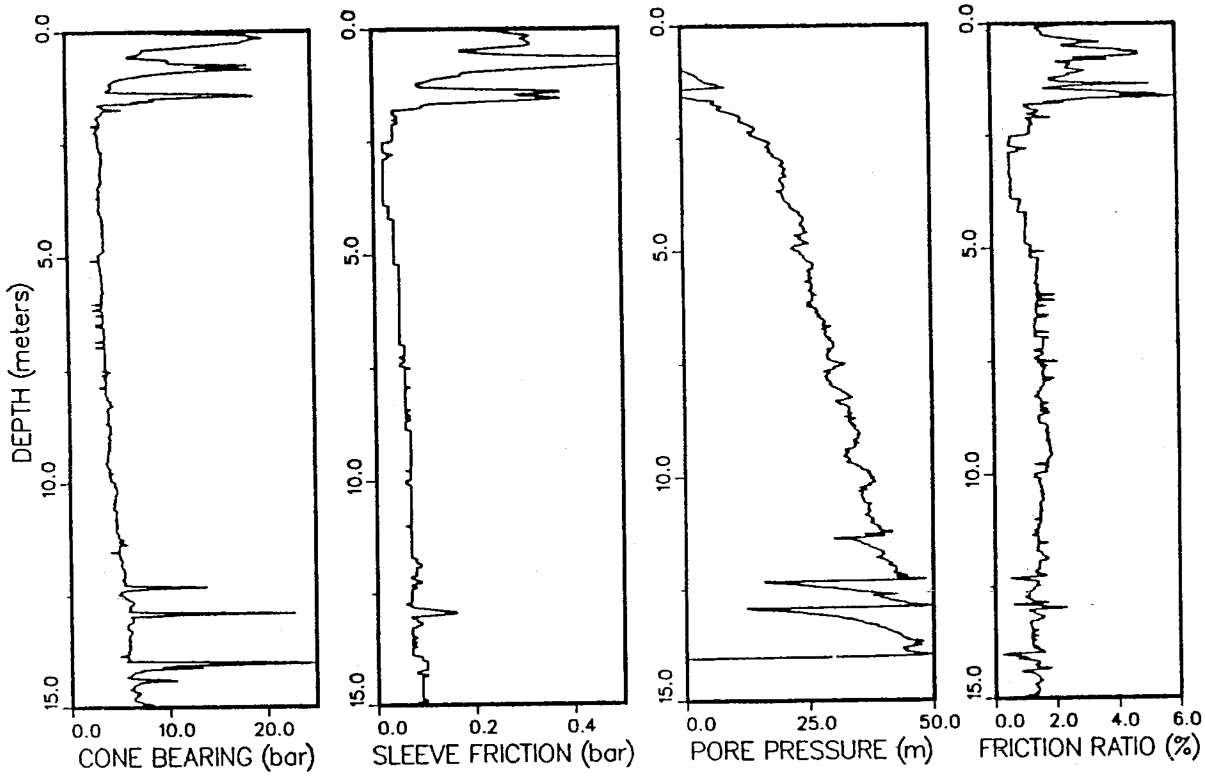

Figure 4: Cone Bearing, Sleeve Friction, Pore Pressure and Friction Ratio Profiles at Langley Site.

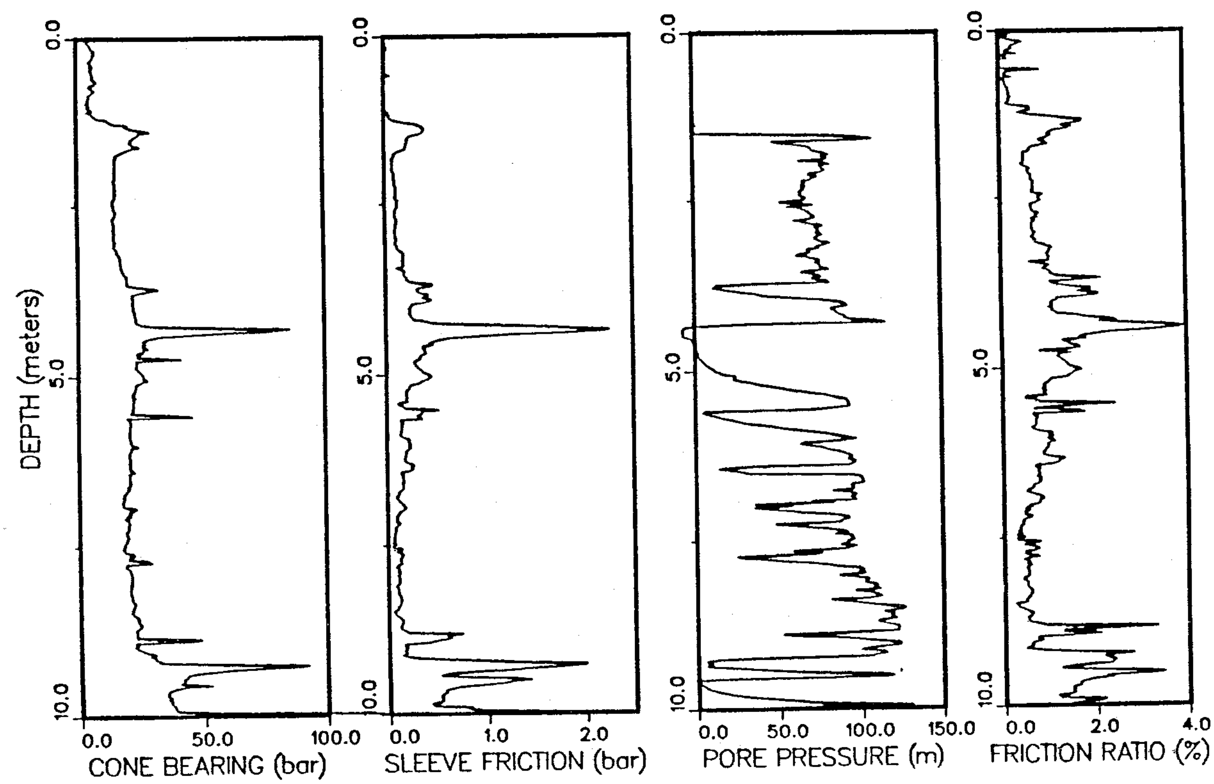

Figure 5: Cone Bearing, Sleeve Friction, Pore Pressure and Friction Ratio Profiles at Strong Pit Site.

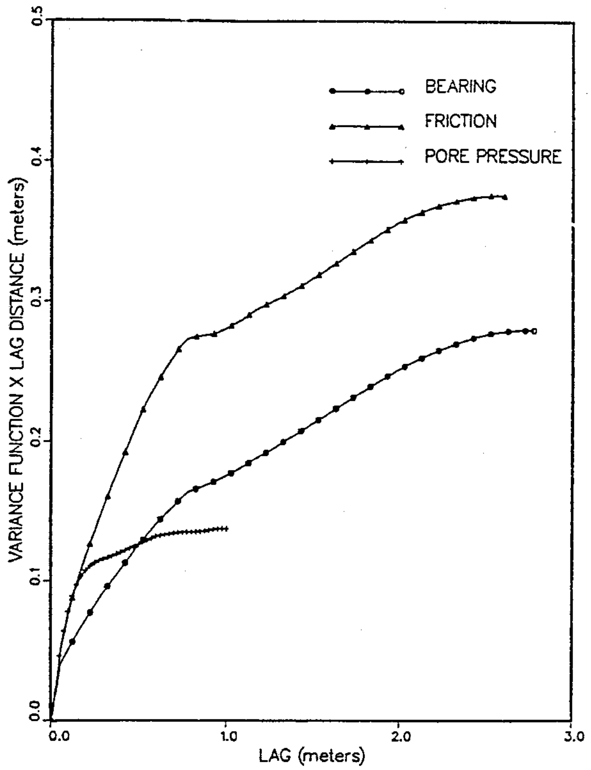

Figure 6: Variation of the Variance Function times Lag Distance for Strong Pit Data.

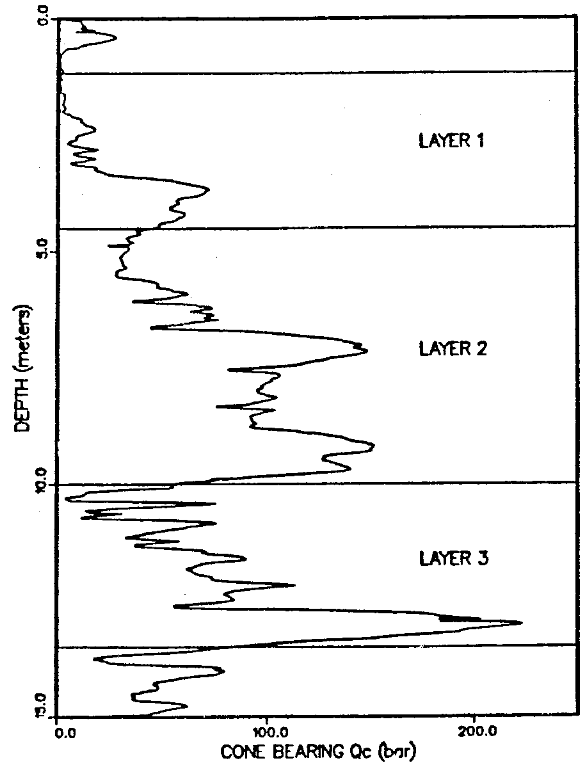

Figure 8: Cone Bearing Profile at McDonald Farm.

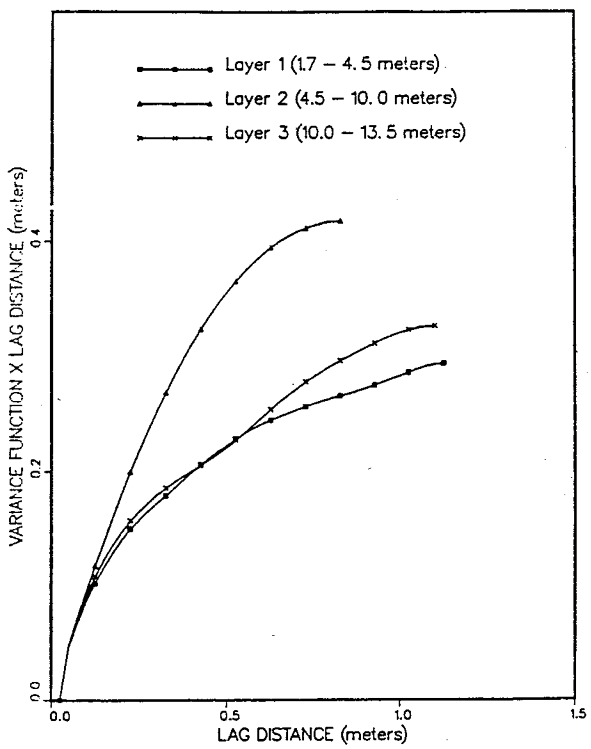

Figure 7: Variation of the Variance Function times Lag Distance for McDonald Farm Data.

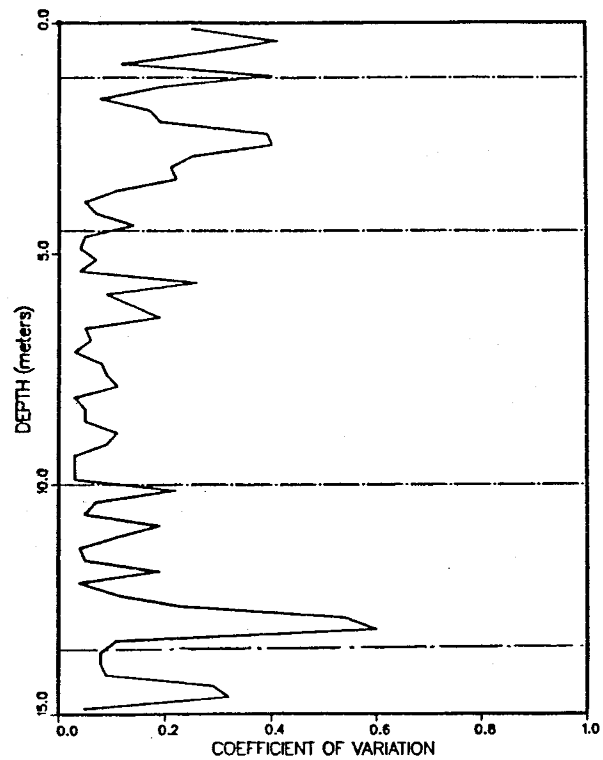

Figure 9: Coefficient of Variation at McDonald Farm.