INFLUENCE OF STRESS PATH ON THE PLANE STRAIN BEHAVIOUR OF A SENSITIVE CLAY

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SYNOPSIS

Results of plane strain tests on a normally consolidated, undisturbed, sensitive, marine clay are presented using a variety of imposed stress paths. For either compression or extension, the undrained behaviour is found to be independent of the total stress path. However, when there is a reversal in the direction of principal stress during shear, for example in extension tests, the undrained strength is reduced. In drained compression, the strain required to cause failure is dependent on the imposed stress path even though the angle of shearing resistance is not. Different peak friction angles are mobilized in compression and extension modes of shear. Also included is a description of the plane strain apparatus which permits K_0-consolidation either in a single increment or under strain controlled loading. The hydraulic loading system allows shear loading under strain controlled conditions even for those stress paths which involve monotonic increase or decrease of lateral fluid pressure. Continuous measurement of side friction is made and suitable corrections applied to the measured vertical stress.

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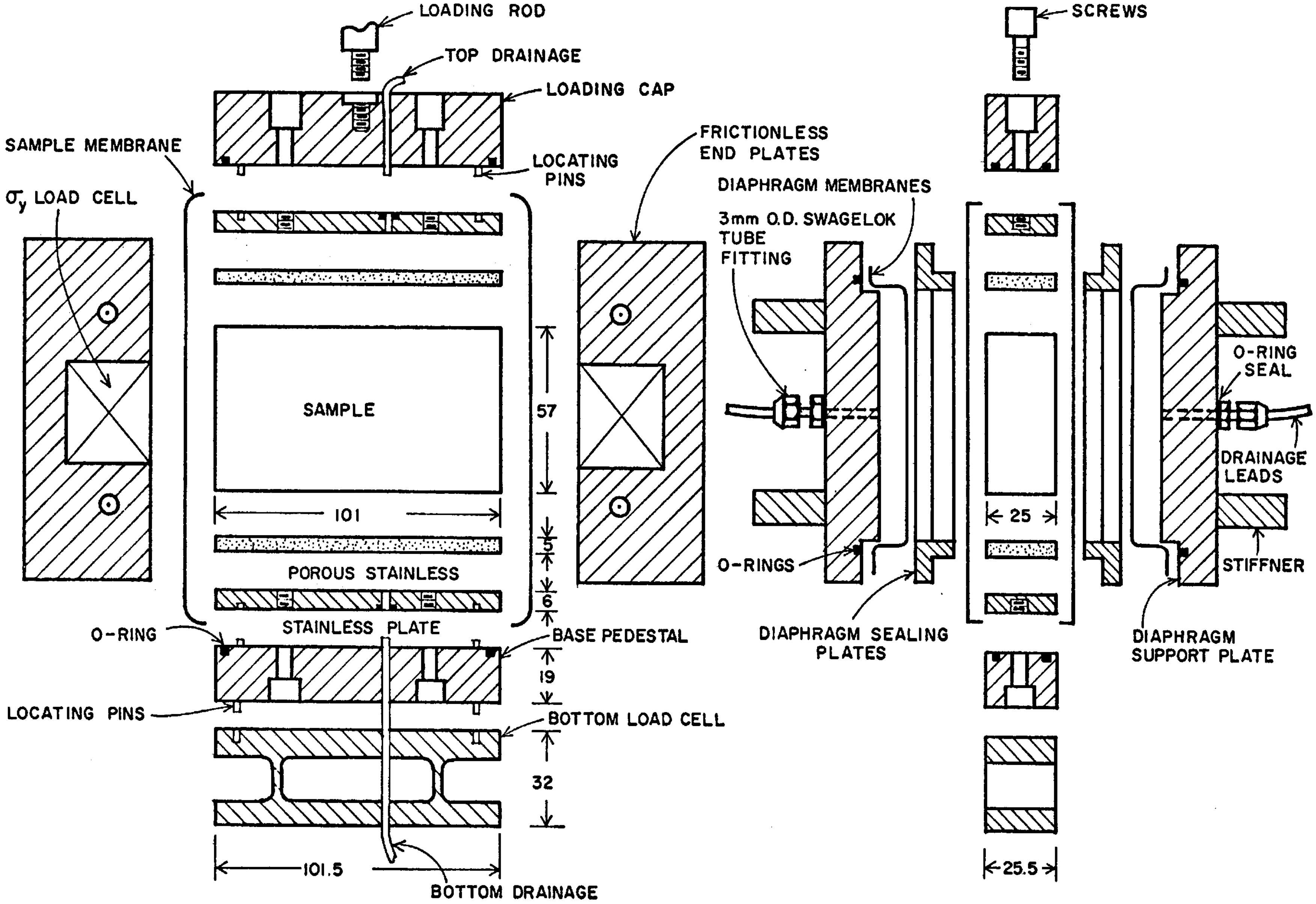
INTRODUCTION

The mechanical properties of most soils depend on the consolidation history and the nature of subsequent stress changes during shear. Consequently, there has been considerable emphasis on the need to determine soil properties from tests in which the consolidation history and subsequent stress changes duplicate those anticipated in a given field problem (Lambe 1967). However, only in a true triaxial apparatus would it be possible to determine the full range of soil behaviour as a function of the state of stress. On the other hand, properties determined under plane strain conditions would suffice for many field problems. In spite of its great practical importance, very little research effort has been spent in the study of the plane strain behaviour of clays after they have been one dimensionally or K_0 consolidated - a condition common to all natural sedimentary deposits. It appears that the only results reported in this field are those by Henkel and Wade (1966) and Hambly and Roscoe (1969). Henkel and Wade, however, restricted themselves to undrained compression behaviour only; whereas, Hambly and Roscoe considered drained and undrained compression and also extension stress paths. Both these studies were confined to remolded clays.

In this paper, the results of plane strain behaviour of an undisturbed, sensitive, marine clay have been presented. Following initial K_0 consolidation, the samples were sheared under a variety of stress paths. This included compression and extension paths under both increasing and decreasing stresses. For each selected stress path, drained and undrained behaviour was studied. The new plane strain apparatus has been briefly described. The prismatic test specimen can be initially K_0 consolidated either incrementally or under strain controlled conditions and all stress paths during shear can be imposed under strain controlled loading. The apparatus was designed to measure all boundary stresses on the sample including side friction.

THE PLANE STRAIN APPARATUS

An exploded view of the plane strain apparatus is shown in Fig. 1. The sample, 10 cm. long, 2.5 cm. wide and initially 5.75 cm. high, is constrained in the direction of its length between two fixed, rigid, smooth end plates, which



NOTE: ALL DIMENSIONS IN MM.

FIG. 1 EXPLODED VIEW OF THE PLANE STRAIN APPARATUS.

ensure the plane strain condition. The end plates are instrumented with load cells in order to measure the normal stress developed. The vertical load is transmitted to the sample by means of a rigid loading cap, whereas a pair of flexible, water filled rubber diaphragms which are sealed to stiffened support plates are employed to furnish the lateral principal stress. The lateral pressure diaphragms and the end plates are positioned around the sample, the loading cap and base pedestal by clamping them together with a set of bolts. In order to prevent the pressurized diaphragms from squeezing out past the opening between the diaphragm support plates and the loading cap or base pedestal, the clearance between the diaphragm support plates and cap or base was kept at 0.015 cm. on each side. Pressures up to 12 kg/cm^2 were developed in these diaphragms which were 0.030 cm. thick and there was no tendency for them to squeeze in between the sample and the end plates or out of the opening between diaphragm support plate and cap or base.

During K_o consolidation, the sample is prevented from straining in the longitudinal direction by the presence of rigid end plates and in the lateral direction by maintaining a constant volume of water in the flexible diaphragms. The very small compliance of the lateral pressure diaphragms does not affect the measured K_o value to any significant extent (Campanella and Vaid 1972).

The vertical load applied to the top of sample is measured on the loading rod. During deformation of the sample in the vertical direction, part of this load is transferred through side friction to the rigid end plates and lateral pressure diaphragms. In order to have a direct measure of this side friction, the base pedestal is instrumented with a load cell which registers the load transmitted to the bottom of the sample. An average vertical stress is thus computed from the loads measured at top and bottom.

The rectangular rubber membrane, which encloses the sample, the top and bottom porous stones, and the stainless sealing plates, has its unstretched linear dimensions about 5% smaller than actual size. The horizontal edges of the membrane are sealed to the top cap and bottom pedestal by means of the stainless sealing plates to which drainage lines are attached. In order to minimize side friction, the membrane is lubricated liberally with silicone grease prior to assembling lateral diaphragms and end plates around the sample.

A schematic layout of the loading system, which has been described in detail elsewhere (Campanella and Vaid 1972), is shown in Fig. 2. The vertical

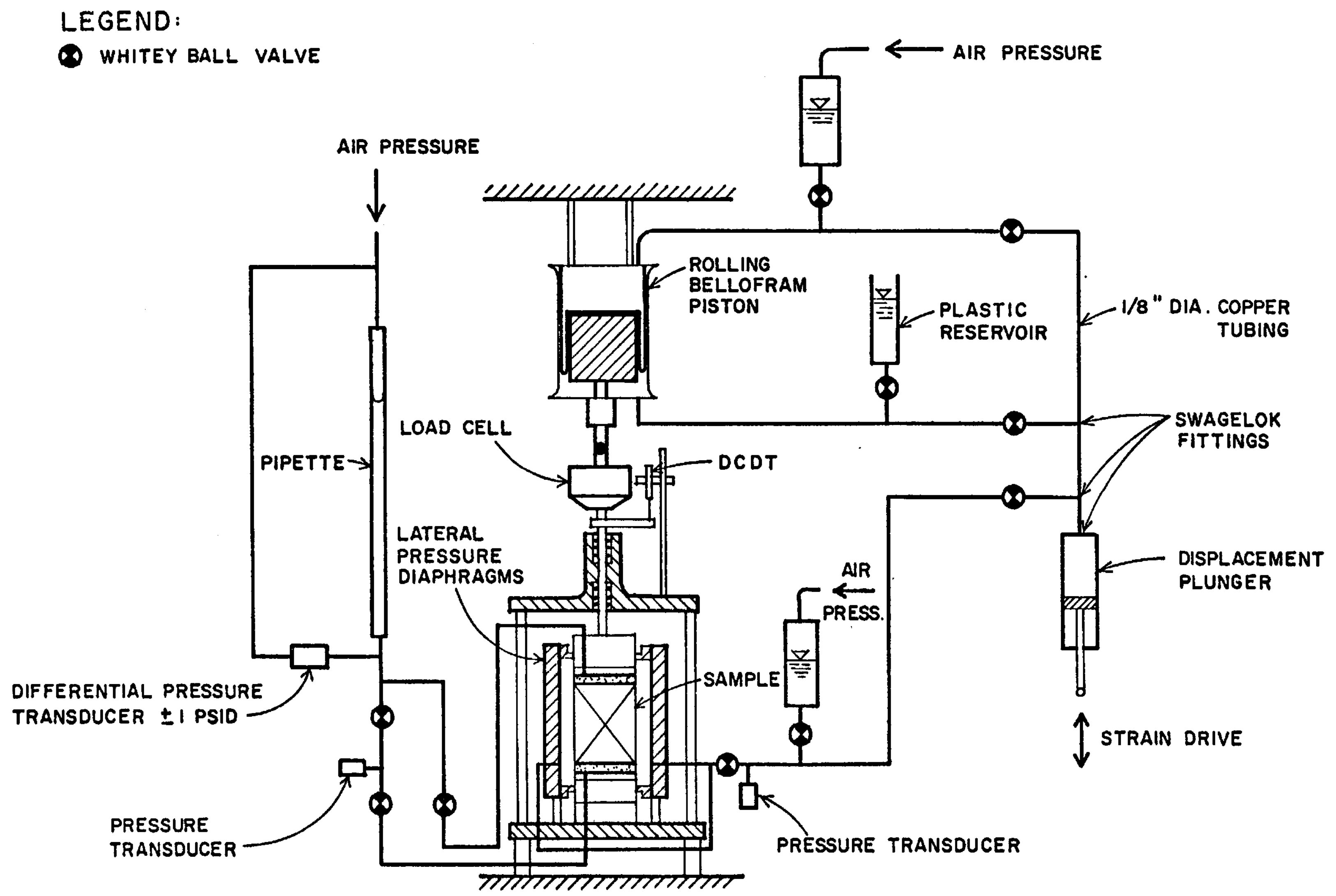


FIG. 2 SCHEMATIC LAYOUT OF LOADING, PRESSURE AND VOLUME CHANGE MEASURING SYSTEM.

load is applied by means of a rolling diaphragm piston which is completely saturated on both sides. Each side of this piston is connected by means of 1/8" O.D. copper tubes to small water reservoirs and also to a displacement plunger actuated by a constant speed drive. The lateral pressure diaphragms are similarly connected to an air pressure supply and to the same displacement plunger.

K_0 consolidation can be performed in either one or several load increments, or under strain controlled loading. For K_0 consolidation in one increment, the sample is loaded undrained with equal vertical and lateral stresses. Pore water drainage is then permitted against a back pressure while the volume of water in the lateral pressure diaphragms is maintained constant. For K_0 consolidation under strain controlled loading, equal vertical and lateral stress is first applied to the undrained sample and then strain controlled loading is commenced while the volume of water in the lateral pressure diaphragm is held constant.

A constant strain rate is imparted to the sample by forcing water from the displacement plunger into the loading piston or into or out of the lateral pressure diaphragms. Conventional compression tests in which the lateral stress is maintained constant are carried out by forcing water into the top of the loading piston, while for extension tests, water is forced into the bottom of the piston. Compression and extension tests in which the vertical stress is held constant and the sample is subjected to a constant lateral strain rate are performed by forcing water out of, or into, the lateral pressure diaphragms. If stress controlled loading is desired, the vertical and lateral stresses are changed in discrete steps to reproduce the predetermined stress path.

TEST DESCRIPTION

The undisturbed, saturated marine clay (Haney clay) used in the study was sampled in blocks from an open pit. Test samples were trimmed from blocks obtained from the same horizon in order to ensure maximum uniformity. The clay has a liquid limit of 44%, plastic limit of 18%, a natural water content from 41 to 43%, and a sensitivity from 6 to 10. Maximum past pressure was found to be about 4 kg/cm^2 from one dimensional consolidation tests.

All samples of clay were normally consolidated under K_o conditions in one increment to the same vertical effective stress of 6 kg/cm^2. Each sample was then subjected to a different stress path in plane strain shear. With reference to Fig. 3, which shows the principal stresses σ_x, σ_y, σ_z and strains ε_x, ε_y, ε_z acting on the sample, the following stress paths were investigated.

Passive compression (P.C.)	- The conventional compression test in which the axial stress σ_z is increased while the lateral stress σ_x is held constant.
Active compression (A.C.)	- Compression test in which the lateral stress σ_x is decreased while the axial stress σ_z is held constant.
Passive extension (P.E.)	- Extension test in which the lateral stress σ_x is increased while the axial stress σ_z is held constant.
Active extension (A.E.)	- Extension test in which the axial stress σ_z is decreased while the lateral stress σ_x is held constant.

For each stress path, drained and undrained tests were carried out. Each test was performed twice and the results were averaged. In drained tests, drainage was permitted from one end of the sample and excess pore pressure, if any, was recorded at the other. Effective stresses were computed assuming a parabolic distribution of this small excess pore pressure. All tests were performed under strain controlled loading.

PLANE STRAIN TEST RESULTS

The existence of identical stress-strain behaviour for a particular normally consolidated clay when normalized with respect to the consolidation stress is well known at least for remolded saturated clay (Hvorslev 1960, Henkel 1960, Roscoe and Poorooshasb, 1963). This was also found to be the case for undisturbed Haney clay. For example, it was found that three passive undrained compression tests on samples, each K_o consolidated to a different vertical

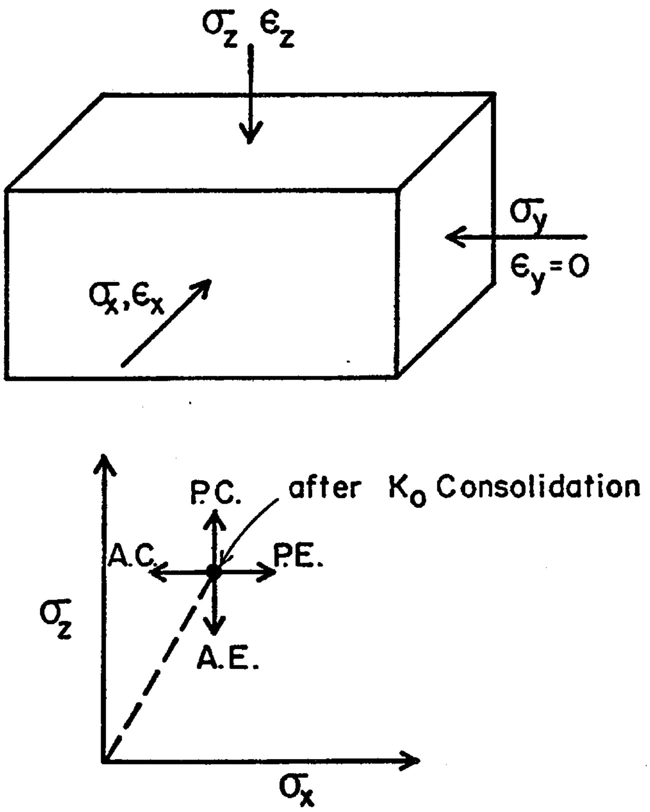


FIG. 3 STRESS PATHS INVESTIGATED

consolidation stress σ'_{zc}, yielded essentially identical normalized deviator stress, $(\sigma_z-\sigma_x)/\sigma'_{zc}$, versus axial strain relationships. It was therefore decided to thoroughly investigate the plane strain behaviour for essentially only one value of normal consolidation stress since behaviour at other values could be obtained by using normalized stress parameters.

Undrained Test Results - The normalized deviator stress, $|\sigma_z-\sigma_x|/\sigma'_{zc}$, versus axial strain, ε_z, behaviour for the four stress paths is shown in Fig. 4. It is interesting to note that the results for the two compression stress paths are identical, as are those for the two extension stress paths. The invariance of undrained stress-strain behaviour with respect to applied total stresses has also been noted in undrained triaxial shear of saturated remolded clays (Henkel 1960, Parry 1960), and is a consequence of shear induced pore pressure being uniquely related to strain.

The Haney clay was very brittle in compression where the axial strain to peak deviator stress was only 0.4%. In contrast, however, the extension failure was quite plastic with a failure strain around 10%. Results of Hambly and Roscoe (1969) on a remolded clay do not show such large differences in strain to failure between the compression and extension modes of shear. The ratio of undrained strength to major consolidation stress, c_u/σ'_{zc}, in extension was approximately 70% of that in compression. It is interesting to note in Fig. 4 that at large strains, the deviator stress in both compression and extension was less than the value the soil could sustain at the end of consolidation when $\varepsilon_z = 0$.

The observed difference between peak strength in compression and extension as well as the fact that these peaks occur at widely different strains suggests that the exclusive use of the compression test to obtain soil parameters may yield unconservative results when used in stability analyses.

A typical characteristic of sensitive clays observed in undrained triaxial compression is the continuous build up of pore pressure after the occurrence of peak deviator stress. Consequently, the condition of peak effective stress ratio is reached with further straining beyond peak of deviator stress. Fig. 5 shows that Haney clay demonstrated similar behaviour in plane strain compression. Peak effective stress ratio was reached at about 4.5% axial strain whereas peak deviator stress occurred at only 0.4% axial strain. In contrast, the strain

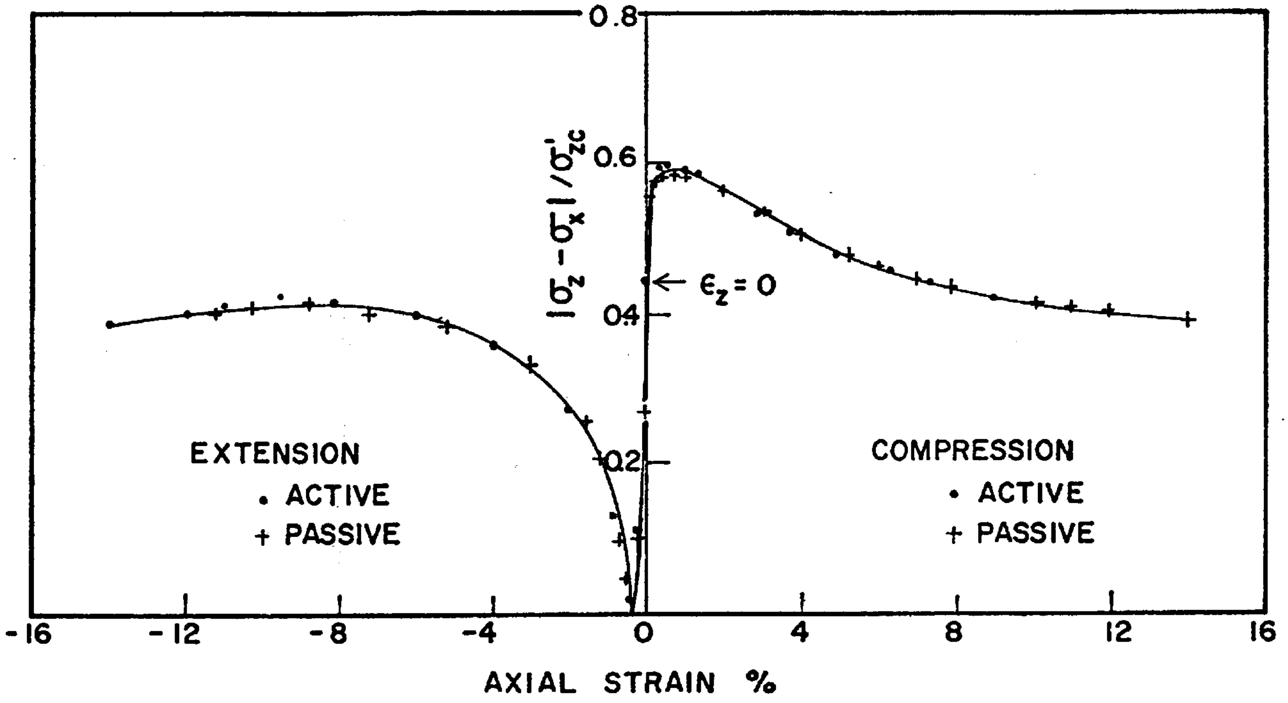


FIG. 4 NORMALIZED DEVIATOR STRESS VERSUS AXIAL STRAIN IN UNDRAINED PLANE STRAIN SHEAR.

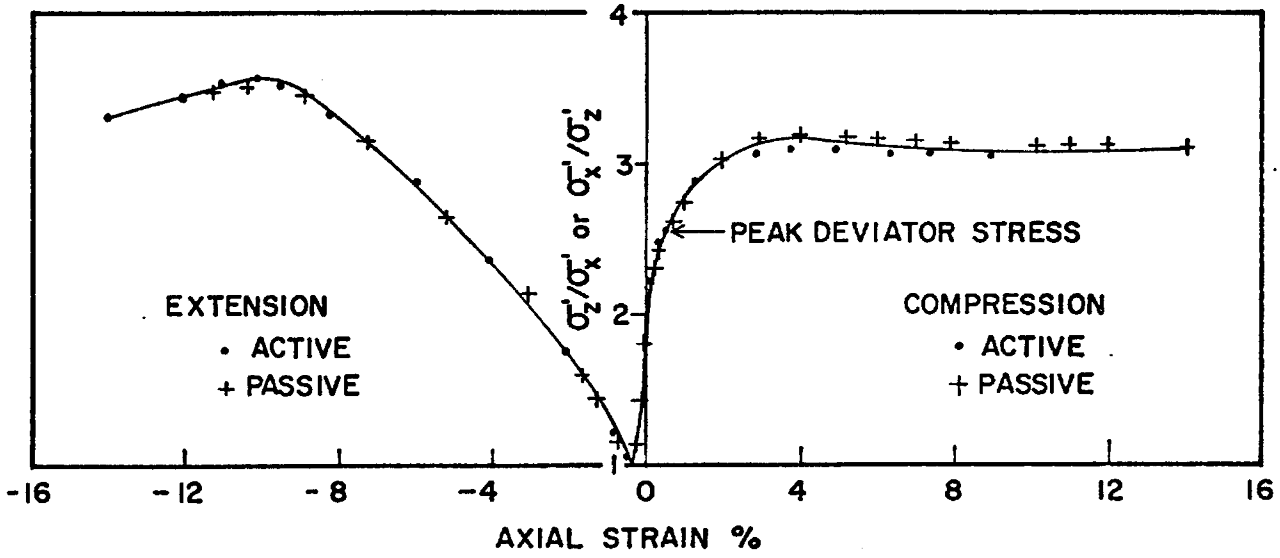


FIG. 5 PRINCIPAL EFFECTIVE STRESS RATIO VERSUS AXIAL STRAIN IN UNDRAINED PLANE STRAIN SHEAR.

at peak deviator stress and peak effective stress ratio was the same for extension shear. The average angle of shear resistance, ϕ', corresponding to the peak effective stress ratio was 31.6 degrees in compression compared to 34.3 degrees in extension. Hambly and Roscoe (1969), however, reported no difference in ϕ' for a remolded clay in plane strain compression and extension.

Drained Test Results - Only three stress paths were studied under drained conditions. A passive extension test could not be carried out due to the limitations on the maximum pressure that could be developed in the lateral pressure diaphragms. Fig. 6 shows the variation of effective stress ratio with axial strain. It is seen in this figure that both compression specimens developed the same peak effective stress ratio and therefore had the same limiting angle of friction, ϕ'. But the strain required to mobilize this ϕ' was about 12% in passive compression compared to only 4% in active compression. Average ϕ' associated with a compression failure was 29.4 degrees compared to 34.7 degrees for extension failure.

The Principal Stress σ'_y - Fig. 7 shows the ratio of principal effective stress σ'_y in the direction of zero strain to the sum of other two principal stresses. Results from all tests are plotted in Fig. 7. Considering the range of stress paths studied, the ratio $\sigma'_y/(\sigma'_x+\sigma'_z)$ stayed remarkably constant during the shearing process and corresponded to a value of between 0.36 and 0.39. Thus as a good approximation σ'_y might be considered to be a constant fraction of $(\sigma'_z+\sigma'_x)$ during plane strain shear of normally consolidated Haney clay. It may be of interest to point out that σ'_y was not always the intermediate principal stress during extension shear, which involved monotonic decrease of effective stress ratio. In the initial stages of the extension shear σ'_y was the minor principal stress. That this must be so can be shown by assuming linear elastic behaviour at small strains.

COMPARISON OF DRAINED AND UNDRAINED RESULTS

Stress-strain comparison - The hypothesis that a unique relationship exists between void ratio and effective stresses during triaxial compression shear of isotropically consolidated samples of saturated remolded clays was first advanced by Rendulic (1937) and later amplified by Henkel (1960). Both Rendulic and Henkel

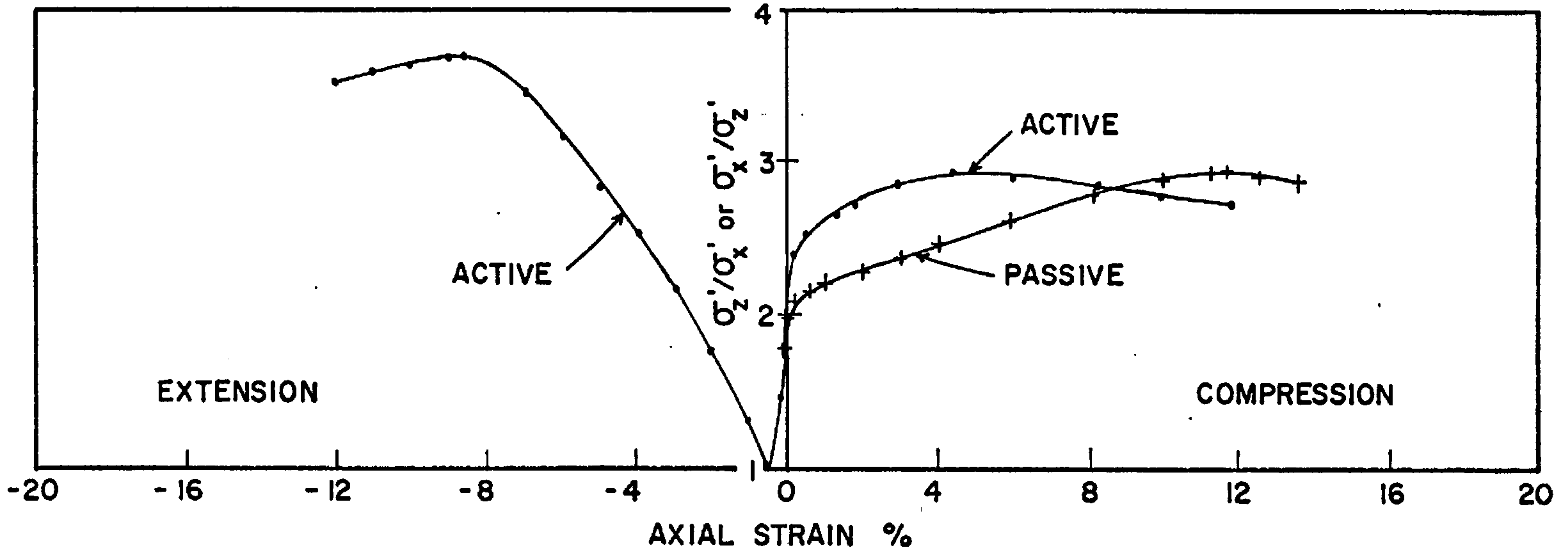


FIG. 6 PRINCIPAL EFFECTIVE STRESS RATIO VERSUS AXIAL STRAIN IN DRAINED PLANE STRAIN SHEAR.

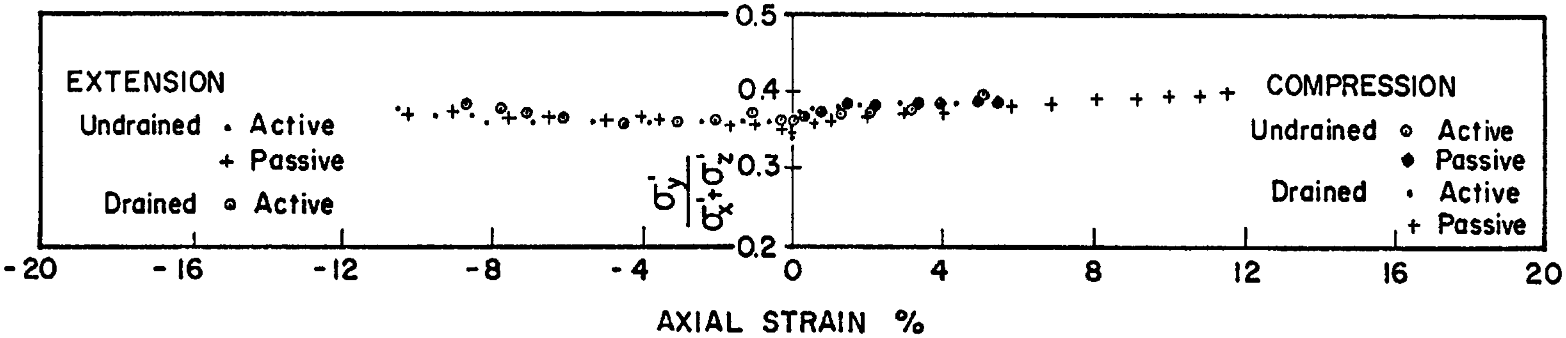


FIG. 7 VARIATION OF STRESS RATIO $\sigma_y'/(\sigma_x'+\sigma_z')$ WITH AXIAL STRAIN IN PLANE STRAIN SHEAR.

suggested that anisotropically consolidated samples would also fit into the same pattern. This suggestion, however, was later found to be incorrect (Henkel and Sowa 1963, Whitman et al 1960, Skempton and Sowa 1963). It was believed that for the hypothesis to be valid, the initial consolidation history of the clay must be the same. No attempt was made to demonstrate that initially K_o consolidated clays might also possess a unique void ratio effective stress relationship during drained and undrained triaxial compression.

Roscoe, Schofield and Wroth's (1958) concept of a unique state boundary surface was an alternate statement of Rendulic's concept of uniqueness of void ratio and effective stresses. This state boundary surface degenerates into a single curve in p/p_e, q/p coordinates (Roscoe and Poorooshasb 1963). Here

$$p = \frac{1}{3}(\sigma_1' + \sigma_2' + \sigma_3')$$

$$q = \sigma_1' - \sigma_3'$$

p_e = Equivalent pressure - which is the pressure on the virgin consolidation line corresponding to the current void ratio e of the sample.

The stress parameters p and q specify completely the current state of stress in the triaxial test. Rendulic's uniqueness concept then implies that normally consolidated saturated remolded clay, with a given consolidation history, when tested in drained or undrained compression possesses a unique surface in (p, q, e) space.

A unique surface in (p, q, e) space implies that void ratio is a function of mean normal effective stress, p, and of shear stress, q, in which $\sigma_2' = \sigma_3'$. During plane strain deformation, however, $\sigma_1' \neq \sigma_2' \neq \sigma_3'$ and therefore octahedral shear stress,

$$\tau_{oct} = \frac{1}{3}\sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2}$$

is a more appropriate measure of shear stress. If it is now postulated that a unique surface in (p, τ_{oct}, e) space exists for plane strain compression tests on K_o consolidated Haney clay, then the implications thereof can be used to correlate drained and undrained tests. The equivalent two-dimensional plot of this surface will be with p/p_e and τ_{oct}/p_e axes, where p_e would now correspond to points on the virgin K_o consolidation line. This curve can be completely determined from undrained compression tests in which p_e is the mean normal consolidation stress. Fig. 8 shows this undrained state boundary surface curve for Haney clay. According to the postulated uniqueness concept, the states of the sample during drained compression will also lie on this curve, and thus it is possible to predict changes in void ratio due to the addition of a stress increment in a drained stress path. For example, for the current state of stress $(p, \tau_{oct})_1$ the ratio τ_{oct}/p can be determined and then from Fig. 8 the point where this stress ratio intersects the undrained state boundary surface curve is found and the value of p/p_e determined. Since p is known, this gives p_e which is the point on the virgin K_o consolidation line having the same void ratio as the stress state $(p, \tau_{oct})_1$. After the stress increment is applied, the new value of p_e is found in the same manner for the new state of stress $(p, \tau_{oct})_2$. Knowing the compression index $\lambda = (e_1 - e_2)/(\ln p_2 - \ln p_1)$ ($\lambda = 0.252$ from one dimensional consolidation tests for Haney clay) the difference in void ratio for the two stress states can be determined, and can be expressed as a volumetric strain.

τ_{oct}/p versus volumetric strain relationships predicted by the above mentioned technique are shown in Figs. 9 and 10 by solid curves for plane strain drained passive and active compression. The observed relationships are shown by data points. Except for small discrepancies in the initial stages of passive compression, there is very good agreement between observed and predicted strains. It is suggested that Rendulic's uniqueness concept could provide a satisfactory means of relating volumetric strains in drained plane strain tests to pore pressures in corresponding undrained tests, provided suitable stress parameters are chosen and the samples have the same consolidation history.

Drained and undrained behaviour under the active extension stress path was essentially identical (compare extension effective stress ratio-axial strain curves in Figs. 4 and 5). The reason for this was the very small volumetric strain during the drained test, thus rendering the situation close to undrained.

Failure conditions - Because of the sensitive nature of Haney clay there were two conditions of failure in undrained shear, ie., one corresponding to peak deviator stress and the other to peak effective stress ratio. Both these conditions have been shown in the summary in Table 1, however, differences between ϕ for deviator stress and peak stress ratio exist only for undrained compression, in which ϕ' was about 6 degrees higher at peak effective stress ratio than

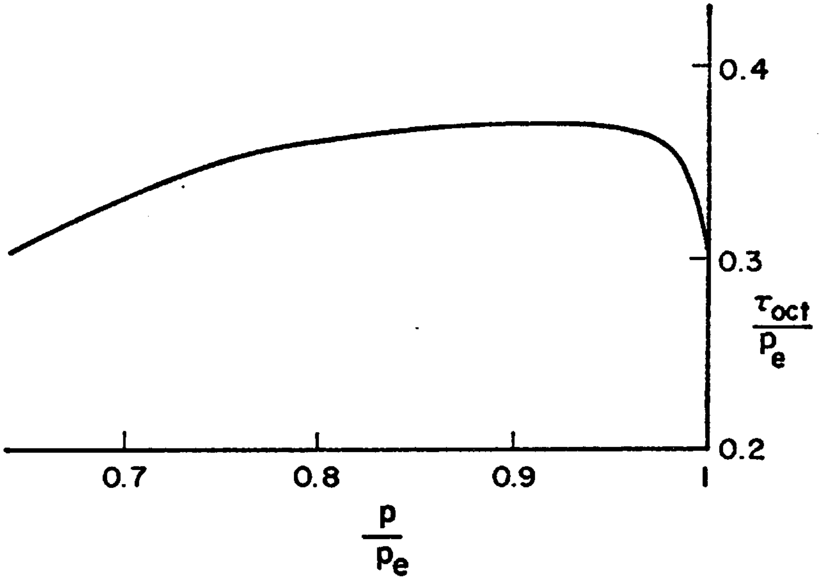


FIG. 8 UNDRAINED STATE BOUNDARY SURFACE CURVE FOR PLANE STRAIN COMPRESSION TESTS.

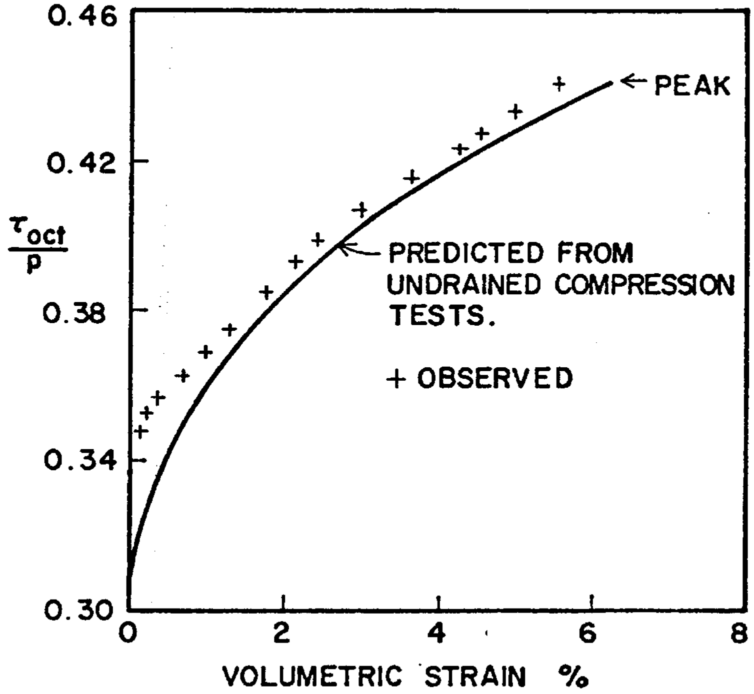


FIG. 9 COMPARISON OF OBSERVED AND PREDICTED VOLUMETRIC STRAINS IN PLANE STRAIN DRAINED PASSIVE COMPRESSION.

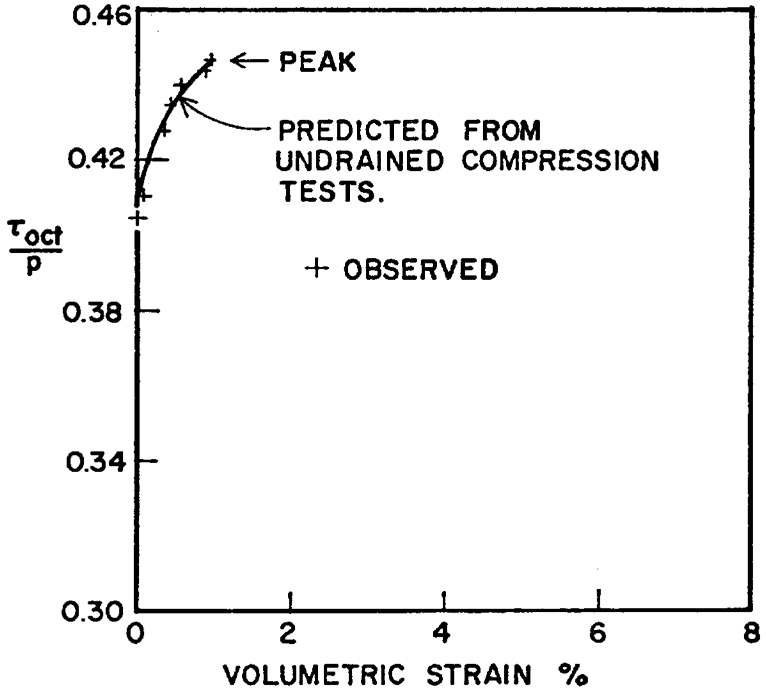


FIG. 10 COMPARISON OF OBSERVED AND PREDICTED VOLUMETRIC STRAINS IN PLANE STRAIN DRAINED ACTIVE COMPRESSION.

TABLE I

Summary of Plane Strain Results on Normally Consolidated
Undisturbed Haney Clay

	Failure Condition								
	$(\sigma_1 - \sigma_3)_{max}$					$(\sigma_1'/\sigma_3')_{max}$			
	Axial Strain %	Vol. Strain %	c_u/σ_{1c}'	σ_1'/σ_3'	ϕ' deg.	Axial Strain %	Vol. Strain %	σ_1'/σ_3'	ϕ' deg.
Undrained Compression	0.4	-	0.296	2.48	25.2	4.5	-	3.20	31.6
Undrained Extension	-10.5	-	0.210	3.57	34.3	-10.5	-	3.57	34.3
Drained:									
Passive Compression	12.0	6.2	-	2.93	29.4	12.0	6.2	2.93	29.4
Active Compression	4.0	0.9	-	2.93	29.4	4.0	0.9	2.93	29.4
Active Extension	-8.6	-0.8	-	3.65	34.7	-8.6	-0.8	3.65	34.7

at peak deviator stress.

When comparing ϕ' values at peak σ_1'/σ_3' it is immediately evident that the values in extension were essentially the same or about 34.5° whether drained or undrained. The values in compression were also essentially the same ranging from about 29.5° for drained to 31.5° for undrained. This range in ϕ' can be directly attributed to different volume change characteristics during shear. Generally speaking it can be concluded that for normally consolidated Haney clay, the shear strength parameter ϕ' in plane strain was an average of 3° higher in extension than compression. A similar order of difference in compression and extension ϕ' for this clay was also found in K_0 triaxial tests (Campanella and Vaid 1972).

CONCLUSIONS

Results of plane strain tests on a normally consolidated, undisturbed, sensitive, marine clay using a variety of imposed stress paths enable the following conclusions to be drawn:

1. For either compression or extension, the undrained response is unaffected by applied total stresses.
2. Interchange of principal stress directions during shear results in a loss of undrained strength.
3. For a given mode of deformation, ie., either compression or extension, the strain to failure in drained shear is dependent on the imposed stress path, even though the peak friction angle mobilized is the same.
4. Peak friction angles are lower by an average of 3° in compression compared to extension failures.
5. Irrespective of the stress path, the principal stress in the direction of zero strain maintains a constant ratio with respect to the sum of the other two principal stresses.
6. A modified form of Rendulic's hypothesis provides a satisfactory method of relating volume changes in drained compression tests with pore pressures in undrained compression tests.

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REFERENCES

Campanella, R.G. and Vaid, Y.P. (1972), "A simple k_o-triaxial cell", Canadian Geotechnical Journal, 9:3.

Hambly, E.C. and Roscoe, K.H. (1969), "Observations and predictions of stresses and strains during plane strain of 'wet' clays", Proc. 7th Intn. Conf., Mexico, 1:173-181.

Henkel, D.J. (1960), "The shear strength of saturated remolded clays", ASCE Research Conf. on the Shear Strength of Cohesive Soils, Boulder, Colorado, 533-554.

Henkel, D.J. and Sowa, V.A. (1963), "The influence of stress history on stress paths in undrained triaxial tests in clay", ASTM STP 361, 280-291.

Henkel, D.J. and Wade, N.H. (1966), "Plane strain tests on a saturated remolded clay", J. Soil Mech. and Found. Div., ASCE, 92:6, 67-80.

Hvorslev, M.J. (1960), "Physical components of shear strength of saturated clays", ASCE Research Conf. on Shear Strength of Cohesive Soils, Boulder, Colorado, 169-273.

Lambe, T.W. (1967), "Stress path method", J. Soil Mech. and Found. Div., ASCE, 93:6, 309-332.

Parry, R.G.H. (1960), "Triaxial compression and extension tests on remolded saturated clay", Geotechnique, 10:4.

Rendulic, L. (1937), "Ein grundgesetz der tonmechanik und sin experiment eller beweiss", Der Bauingenieur, 18, 459-467.

Roscoe, K.H. and Poorooshasb, H.B. (1963), "A theoretical and experimental study of strains in triaxial tests on normally consolidated clay", Geotechnique, 13:12-38.

Roscoe, K.H., Schofield, A.N. and Wroth, P.W. (1958), "On the yielding of soils", Geotechnique, 8:22-53.

Skempton, A.W. and Sowa, V.A., "The behaviour of saturated clay during sampling and testing", Geotechnique, 13:269-290.

Whitman, R.V., Ladd, C.C. and Cruz, P. da. (1960), "Discussion, session 3", ASCE Research Conf. on Shear Strength of Cohesive Soils, Boulder, Colorado, 1049-1056.