

SOIL CREEP AS A RATE PROCESS  
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By

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ABSTRACT

The fundamental equation of the theory of absolute reaction rates has been developed and adapted for use in the study of time-dependent soil deformations. The results of constant stress creep tests have established that the basic form of this equation is supported as regards the assumption of a thermally activated creep process, the predicted relationship between experimental activation energy and creep stress, and the predicted stress dependence of creep rate. Some modification of the equation is needed, however, to account for the variation of creep rate with time.

Techniques are described for study of deformation mechanisms in soils through evaluation of the activation energy for creep and the number of interparticle bonds per unit area from the results of constant stress creep tests. Test results suggest that interparticle bonding is probably of the primary valence type, the resistance to shear stresses cannot be accounted for in terms of viscous water films, and interparticle contacts must be effectively solid to solid.

#### KEY WORDS

activation energy, adsorbed water, cohesive soils, creep tests,  
deformation mechanism, interparticle bonding, plastic deformation,  
rate processes, rheology, shear, shear apparatus, statistical  
mechanics, strain rate, temperature effects, triaxial compression,  
undrained shear tests.

## NOTATIONS

The following symbols are used in this paper.

- A = strain rate at time  $t_1$  and  $D = 0$
- D = creep deviator stress
- E = experimental activation energy, calories/mole
- f = shear force acting on a flow unit
- h = Planck's constant =  $6.624 \times 10^{-27}$  erg-sec<sup>-1</sup>
- K =  $2X \frac{kT}{h} \exp \left[ -\frac{\Delta F}{RT} \right]$
- k = Boltzmann's constant =  $1.38 \times 10^{-16}$  erg °K<sup>-1</sup>
- m = negative of slope of the relationship between logarithm of strain rate and logarithm of time
- R = universal gas constant =  $1.98$  cal °K<sup>-1</sup> mole<sup>-1</sup>
- S = number of flow units
- T = absolute temperature, °K
- t = time
- $t_1$  = reference time
- X = parameter relating activation frequency to strain rate
- $\alpha = \frac{\lambda}{4SkT}$
- $\alpha' = \frac{\lambda}{2kT}$
- $\Delta F$  = free energy of activation, calories/mole
- $\dot{\epsilon}$  = strain rate
- $\lambda$  = distance between equilibrium positions of flow units
- $\nu$  = frequency of activation, sec<sup>-1</sup>

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### INTRODUCTION

The theory of absolute reaction rates, known more commonly as rate process theory, formulated originally by Eyring (Glasstone, Laidler, and Eyring, 1941),<sup>4</sup> has been used recently by several investigators to describe the creep and consolidation behavior of clays under stress (Murayama and Shibata, 1958, 1961, 1964;<sup>5,6,7</sup> Mitchell, 1964;<sup>8</sup>

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<sup>4</sup> Glasstone, S., Laidler, K. and Eyring, H. (1941), The Theory of Rate Process, McGraw-Hill, 1941.

<sup>5</sup> Murayama, S. and Shibata, T. (1958), "On the Rheological Characteristic of Clay," Part I, Bulletin No. 26, Disaster Prevention Research Inst., Kyoto University, Japan, 1958.

<sup>6</sup> Murayama, S. and Shibata, T. (1961), "Rheological Properties of Clays," Proceedings - 5th Int. Congress Soil Mech. and Found. Eng., pp. 269-273, 1961.

<sup>7</sup> Murayama, S. and Shibata, T. (1966), "Flow and Stress Relaxation of Clays (Theoretical Studies on the Rheological Properties of Clay - Part 1)" Rheology and Soil Mech. Symposium of the Int. Union of Rheoretical and Applied Mechanics, held in Grenoble, April, 1964.

<sup>8</sup> Mitchell, J. K. (1964), "Shearing Resistance of Soils as a Rate Process," Journal of Soil Mech. and Found. Div., Amer. Soc. of Civil Engineers, Vol. 90, No. SMI, Proceedings Paper 3773, pp. 29-61, Jan. 1964.

Christensen and Wu, 1964,<sup>9</sup> Wu, Resendiz, and Neukirchner, 1966;<sup>10</sup> Abdel-Hady and Herrin, 1966<sup>11</sup>). Since this theory can be applied to any process involving the time-dependent rearrangement of matter it has the potential for providing a powerful tool for the description and prediction of soil behavior.

It should be noted, however, that thus far most investigations based on rate process theory have assumed a priori that the basic premises of this theory are valid when applied to soil masses. Rheologic models have been proposed (Murayama and Shibata, 1958;<sup>5</sup> Christensen and Wu, 1964;<sup>9</sup> Abdel-Hady and Herrin, 1966<sup>11</sup>), incorporating dashpots whose parameters are developed from rate process theory, which give a reasonably accurate description of the creep, stress relaxation, and consolidation behavior of some clays under some range of loadings. This has been taken as evidence that the basic premises of the theory are correct. Unfortunately, it is possible that the correlation may be due only to the mathematical consequences of the arbitrarily chosen arrangement of springs and dashpots.

Furthermore it would appear that the simple phenomenological relationships found by Singh and Mitchell (1967)<sup>12</sup> to hold for the creep behavior of a variety of clays, and illustrated in Figs. 1 and 2 for constant stress

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<sup>9</sup>Christensen, R. W. and Wu, T. H. (1964), "Analysis of Clay Deformation as a Rate Process," Proc. 4147, Journal of Soil Mech. and Found. Eng. Div., Amer. Soc. of Civil Engineers, pp. 125-157, November, 1964.

<sup>10</sup>Wu, T. H., Resendiz, D. and Neukirchner, R. J. (1966) "Analysis of Consolidation by Rate Process Theory," Journal of the Soil Mechanics and Foundations Division, Proc. Am. Soc. of Civil Engineers, November, 1966.

<sup>11</sup>Abdel-Hady, M. and Herrin, M. (1966) "Characteristics of Soil-Asphalt as a Rate Process," Journal of the highway Division, Proc. Am. Soc. of Civil Engineers, March, 1966.

<sup>12</sup>Singh, A. and Mitchell, J. K. (1967) "A General Stress-Strain Time Function for Soils," in press.

creep tests on undisturbed San Francisco bay mud, cannot be predicted using simple combinations of linear springs and non-linear dashpots derived from rate process theory. Similarly the fact that a number of characteristics of the shearing resistance of soils (Mitchell, 1964)<sup>8</sup> appear to conform to the theory does not provide proof of its validity.

On the other hand, insofar as the authors are aware no rigorous proof of the correctness of the detailed statistical mechanical foundation of the theory has been obtained for even the simplest of chemical reactions. Real behavior in such systems has been found to be substantially in accord with the theoretical predictions, however, and therefore the theory has proven invaluable for the study of many phenomena. Recently Andersland and Akili (1967)<sup>13</sup> have presented data to show that the creep of frozen soils can be treated as a thermally activated process and that the form of the stress dependence of creep rate is in good agreement with predictions of rate theory.

Studies of the creep behavior of soils have been in progress for some time at the University of California. The primary objective of this work has been to develop information relating to the fundamental mechanisms of deformation and the nature of interparticle bonding in soils. Rate process theory has been used as the framework within which much of the work has been carried out. In the course of these studies data have been obtained which serve to lend confidence to the application of the theory to studies of deformation in soils. In this

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<sup>13</sup> Andersland, O. B. and Akili, N. (1967) "Stress Effect on Creep Rates of a Frozen Clay Soil," *Geotechnique*, Vol. 17, pp. 27-39, March 1967.

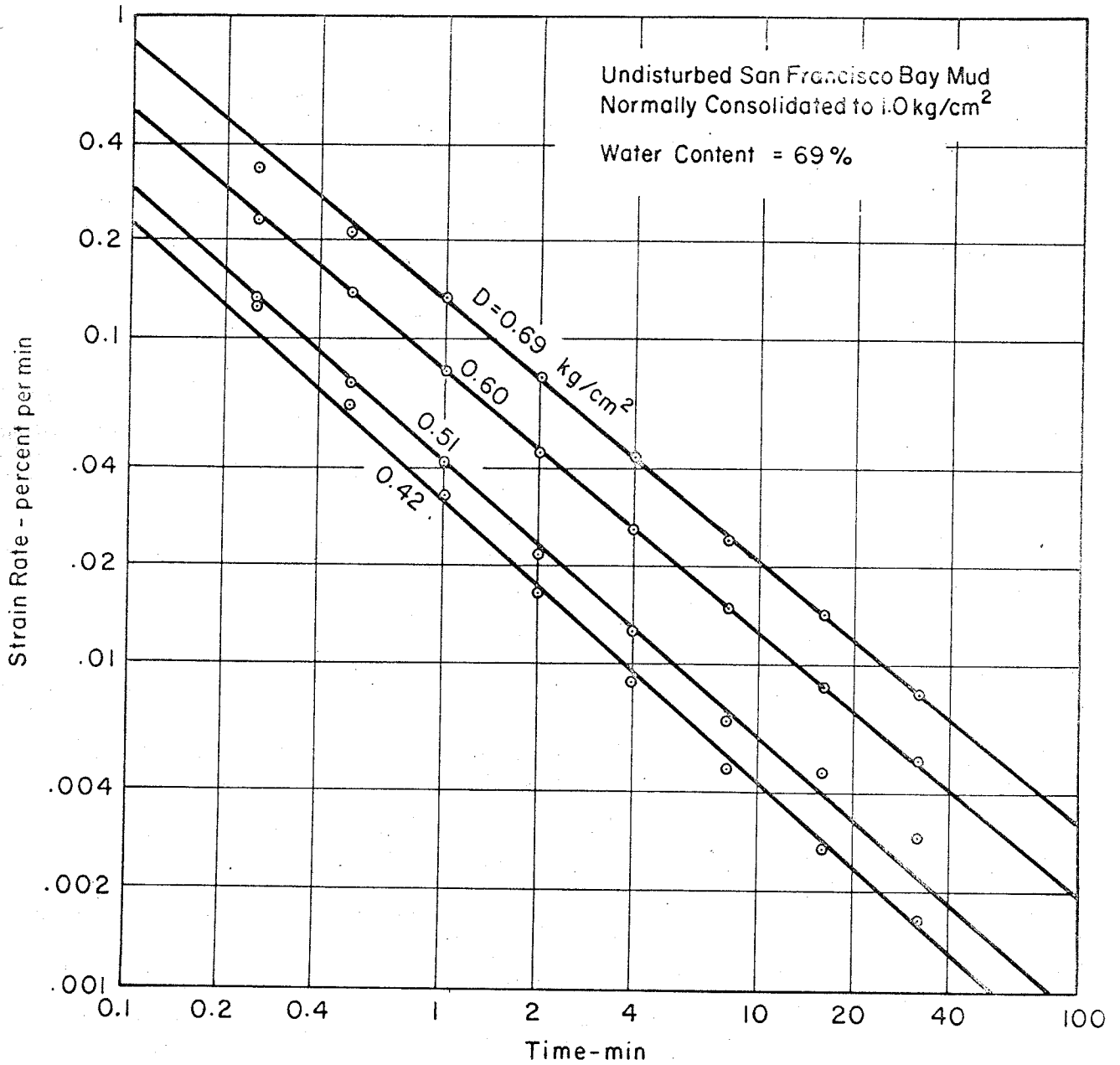


FIG. 1 STRAIN RATE VERSUS TIME RELATIONSHIPS DURING UNDRAINED CREEP OF UNDISTURBED SAN FRANCISCO BAY MUD

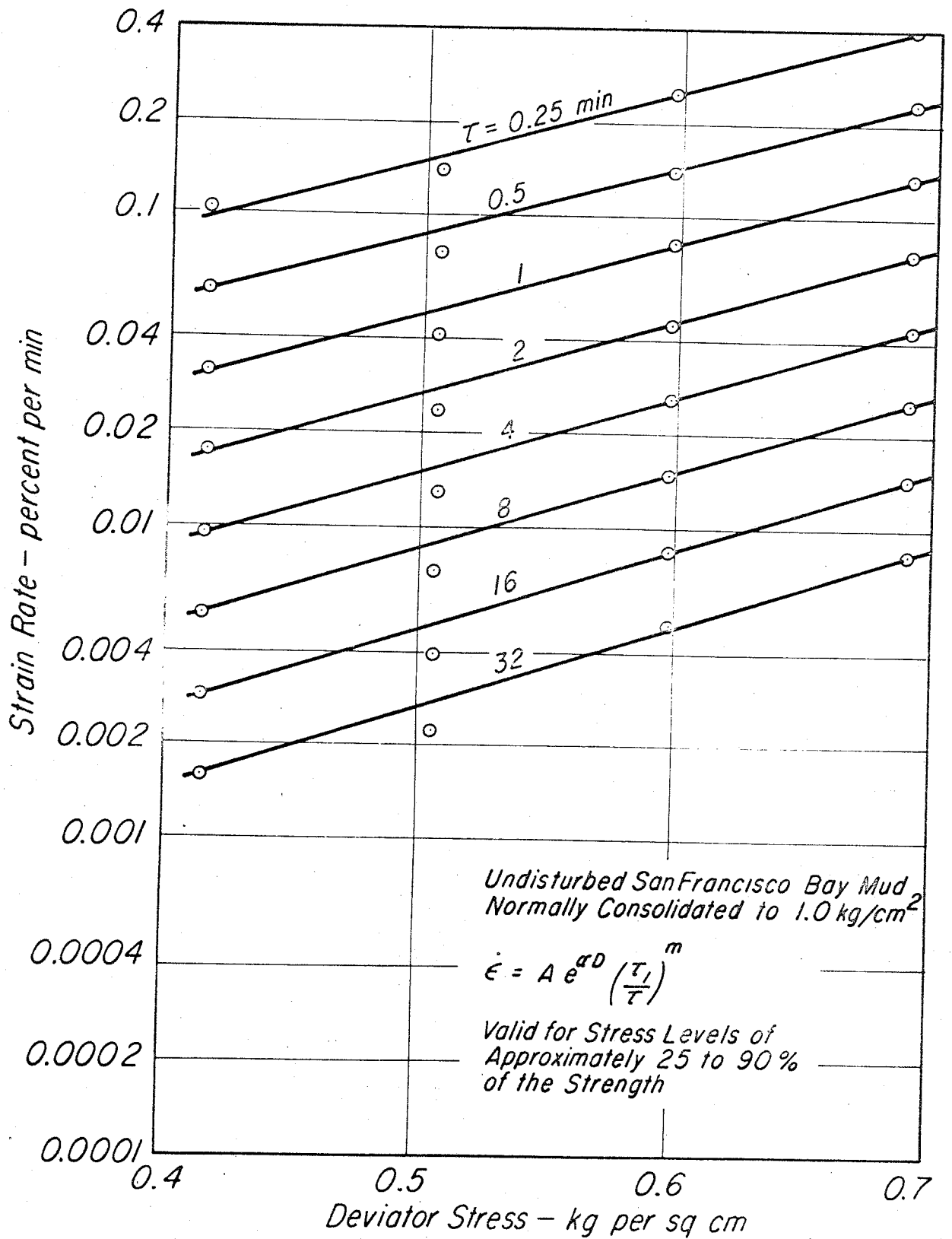


FIG.2 STRAIN RATE VERSUS STRESS RELATIONSHIPS FOR UNDRAINED CREEP OF UNDISTURBED SAN FRANCISCO BAY MUD

paper these data are reviewed, methods for studying creep mechanisms are described, and some tentative interpretations concerning inter-particle bonding in soils are presented.

#### THEORETICAL CONSIDERATIONS

The derivation of the basic rate process equation is given in many references and will not be repeated in detail herein (Eyring, 1936;<sup>14</sup> Glasstone, Laidler and Eyring, 1941;<sup>4</sup> Murayama and Shibata, 1958;<sup>5</sup> Mitchell, 1964;<sup>8</sup> Christensen and Wu, 1964).<sup>9</sup> The basis of the relationship is that the atoms and molecules participating in a deformation process (termed flow units) are constrained from movement relative to each other by virtue of energy barriers separating adjacent equilibrium positions as depicted schematically by curve A in Fig. 3. The displacement of flow units to new positions requires that they become "activated" through acquisition of sufficient energy,  $\Delta F$ , to surmount the energy barrier.  $\Delta F$  is commonly termed the free energy of activation. From considerations of statistical mechanics it is known that the flow units are in fact not at rest but vibrate with a frequency of  $\frac{kT}{h}$ , as a consequence of their thermal energy, where  $k$  is Boltzmann's constant ( $1.38 \times 10^{-16}$  erg -  $^{\circ}\text{K}^{-1}$ ),  $h$  is Planck's constant ( $6.624 \times 10^{-27}$  erg-sec $^{-1}$ ), and  $T$  is the absolute temperature,  $^{\circ}\text{K}$ . The division of thermal energies among the flow units is given by a Boltzmann distribution, and the specific frequency of activation,  $\nu$ , may be shown to be

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<sup>14</sup>Eyring, H. (1936) "Viscosity, Plasticity, and Diffusion as Examples of Absolute Reaction Rates," Journal of Chemical Physics, Vol. No. 4, pp. 283-291, April, 1936.



$$v = \frac{kT}{h} e^{-\Delta F/RT} \quad (1)$$

where R is the universal gas constant = 1.98 cal °K<sup>-1</sup> mole<sup>-1</sup>.

In the absence of applied potentials to the material no consequences of the periodic activation are observed since barriers will be crossed with equal frequency in all directions. If, however, a directed potential such as a shear stress is applied to the material, then the barrier heights become distorted as shown in Fig. 3, curve B. If  $f$  represents the force acting on a flow unit then the barrier height is reduced an amount  $\frac{f\lambda}{2}$  in the direction of the force and raised a like amount in the direction opposite to the force. The distance  $\lambda$  represents the distance between successive equilibrium positions. Minimums in the energy curve are shown displaced a distance  $\delta$  from their original positions. This represents elastic distortion of the material structure.

Since the barrier height in the direction of the force becomes  $(\Delta F - \frac{f\lambda}{2})$  and in the direction opposite to the force  $(\Delta F + \frac{f\lambda}{2})$ , the net frequency of activation in the direction of the force becomes

$$\vec{v} - \overleftarrow{v} = 2 \frac{kT}{h} \exp\left(-\frac{\Delta F}{RT}\right) \sinh\left(\frac{f\lambda}{2kT}\right) \quad (2)$$

If a parameter X is defined which is a function of the number of flow units in the direction of deformation and the average component of displacement in the same direction due to a single surmounting of the barrier, then the total displacement per unit time will be  $X(\vec{v} - \overleftarrow{v})$  and correspond to the rate of strain,  $\dot{\epsilon}$ .

Thus

$$\dot{\epsilon} = 2X \frac{kT}{h} \exp\left(-\frac{\Delta F}{RT}\right) \sinh\left(\frac{f\lambda}{2kT}\right) \quad (3)$$

As will be shown subsequently the value of  $\left(\frac{f\lambda}{2kT}\right)$  will generally be greater than 1 for stresses sufficient to cause significant creep deformations in most soils, and, therefore,

$$\sinh\left(\frac{f\lambda}{2kT}\right) \sim \frac{1}{2} \exp\left(\frac{f\lambda}{2kT}\right) \quad (4)$$

and (3) becomes

$$\dot{\epsilon} = X \frac{kT}{h} \exp\left(-\frac{\Delta F}{RT}\right) \exp\left(\frac{f\lambda}{2kT}\right) \quad (5)$$

It should be noted that the parameter X may be both time and structure dependent, and evidence to support this contention is presented later.

It is possible to test separately different parts of equation (5) in order to determine its applicability to the study of deformation in soils. A number of relevant tests have been made and the test data have been examined directly, without recourse to artificial rheological models.

#### CREEP AS AN ACTIVATED PROCESS

One test of equation (5) is to examine the response of the creep rate of a soil to variations in temperature. Noting that the gas constant, R, and Boltzmann's constant, k, are related according to

$$k = \frac{R}{N} \quad (6)$$

where N is Avogadro's number ( $6.02 \times 10^{23}$ ), equation (5) can be written

$$\dot{\epsilon} = X \frac{kT}{h} \exp\left[-\frac{\left(\Delta F - \frac{f\lambda N}{2}\right)}{RT}\right] \quad (7)$$

or

$$\dot{\epsilon} = X \frac{kT}{h} \exp\left(-\frac{E}{RT}\right) \quad (8)$$

where E is the experimental activation energy, given by

$$E = \Delta F - \frac{f\lambda N}{2} \quad (9)$$

If X and E are independent of temperature, then from eq. (8)

$$\frac{\partial \ln \left( \frac{\dot{\epsilon}}{T} \right)}{\partial \left( \frac{1}{T} \right)} = - \frac{E}{R} \quad (10)$$

Thus if identical specimens are subjected to creep tests under the same stress intensity but at different temperatures, then there should be a linear variation between  $\left( \frac{\dot{\epsilon}}{T} \right)$  and  $\frac{1}{T}$  having a slope of  $- E/R$ .

In order to test this prediction a series of constant stress creep tests were run on identical samples using the controlled-temperature triaxial apparatus described by Mitchell and Campanella (1964).<sup>15</sup> An obvious procedure might be to test identical specimens under the same creep stress but at different temperatures. This type of test is impractical, however, for the following reasons. Since strain rates change rapidly with time after the start of creep it is likely that structure and effective stress also change rapidly, and these variations may be different for different temperatures. Thus identical structures are only likely to exist at the instant the creep process is initiated, and even in this case there is some question that samples having different temperatures could have the same structure. Furthermore strain rates are very difficult to determine accurately at the instant of load application.

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<sup>15</sup> Mitchell, J. K. and Campanella, R. G. (1964), "Creep Studies on Saturated Clays," Symposium on Laboratory Shear Testing of Soils, ASTM-NRC, Ottawa, Canada, ASTM Special Tech. Publication No. 361, Sept. 1963.

In order to overcome these difficulties and to enable determination of creep rates at two temperatures under conditions of essentially the same structure, thus minimizing effects of variations in  $X$  and  $E$  itself with time of creep, use was made of Dorn's Method. This method was originally proposed for the determination of the activation energy for creep of metals (Tietz and Dorn, 1956).<sup>16</sup> By this procedure the specimen is permitted to creep under a constant deviator stress,  $D_1$ , at temperature,  $T_1$ . The temperature is then increased rapidly to  $T_2$  while the deviator stress is maintained constant. The strain versus time behavior for this type of test is as shown in Fig. 4 for undisturbed San Francisco Bay Mud. A comparison can then be made between the strain rate  $\dot{\epsilon}_1$  pertaining to temperature  $T_1$  at the instant just before the temperature change and  $\dot{\epsilon}_2$  for temperature  $T_2$  at the instant after the temperature change.

Since a temperature change cannot in fact be made instantaneously but requires an adjustment period of about 20 minutes for attainment of new equilibrium in the apparatus used for these tests, an extrapolation procedure was used to find  $\dot{\epsilon}_2$ . The results of many tests have shown that the relationship between logarithm of strain rate and strain is nearly linear both before and after the temperature change at least for short time periods, as shown by Fig. 5. In this figure strain rates are shown for the test results given in Fig. 4. The value of  $\dot{\epsilon}_2$  is easily determined by extrapolation of the log strain rate vs. strain curve for temperature  $T_2$  to the value of strain existing at the instant of temperature change.

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<sup>16</sup>Tietz, T. and Dorn, J. (1956) "Creep of Copper at Intermediate Temperatures," Transactions, Amer. Inst. Mech. Eng., Vol. 206, p. 156.

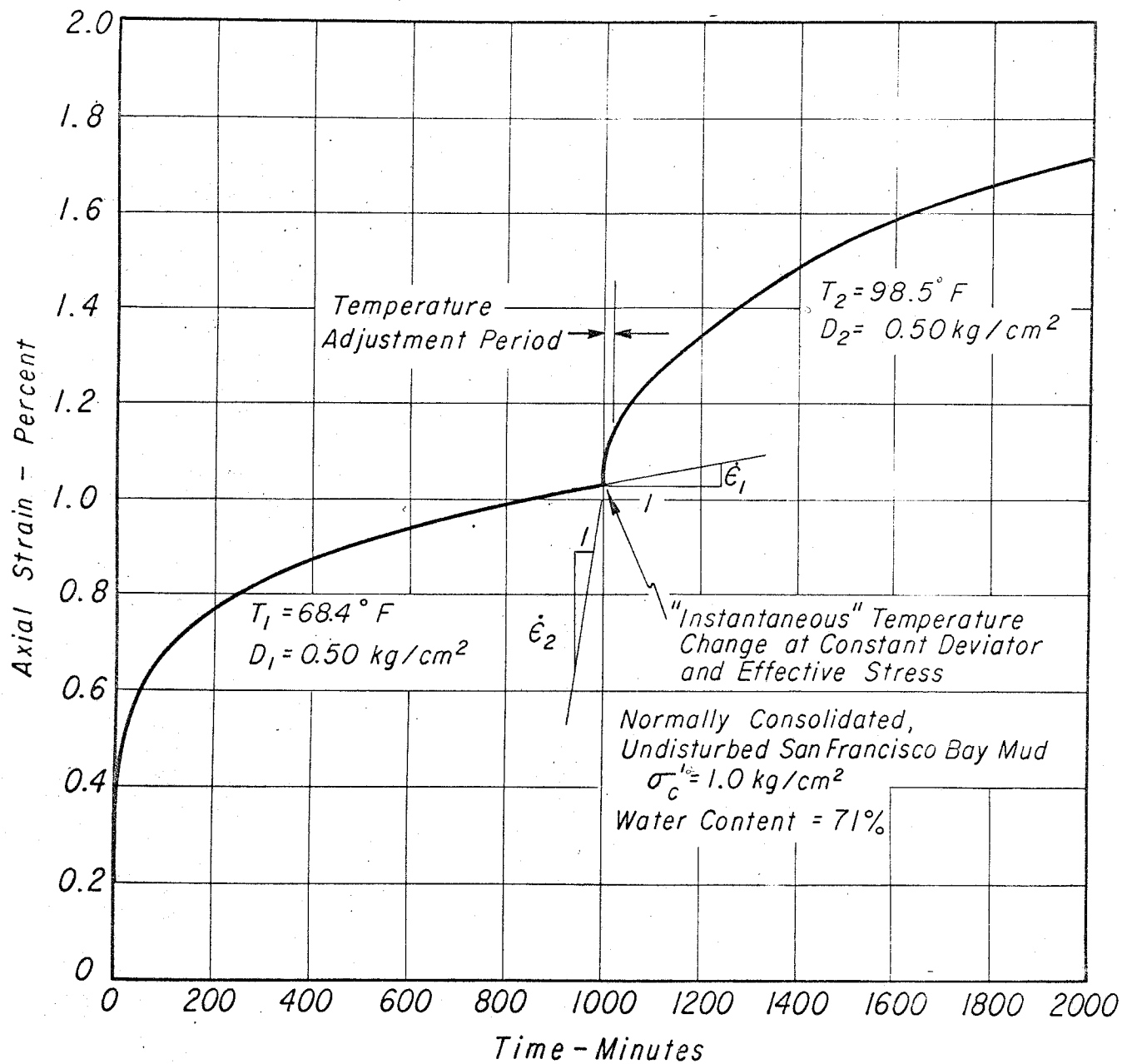


FIG. 4 AXIAL STRAIN VERSUS TIME FOR CREEP TEST WITH RAPID TEMPERATURE CHANGE-UNDISTURBED SAN FRANCISCO BAY MUD

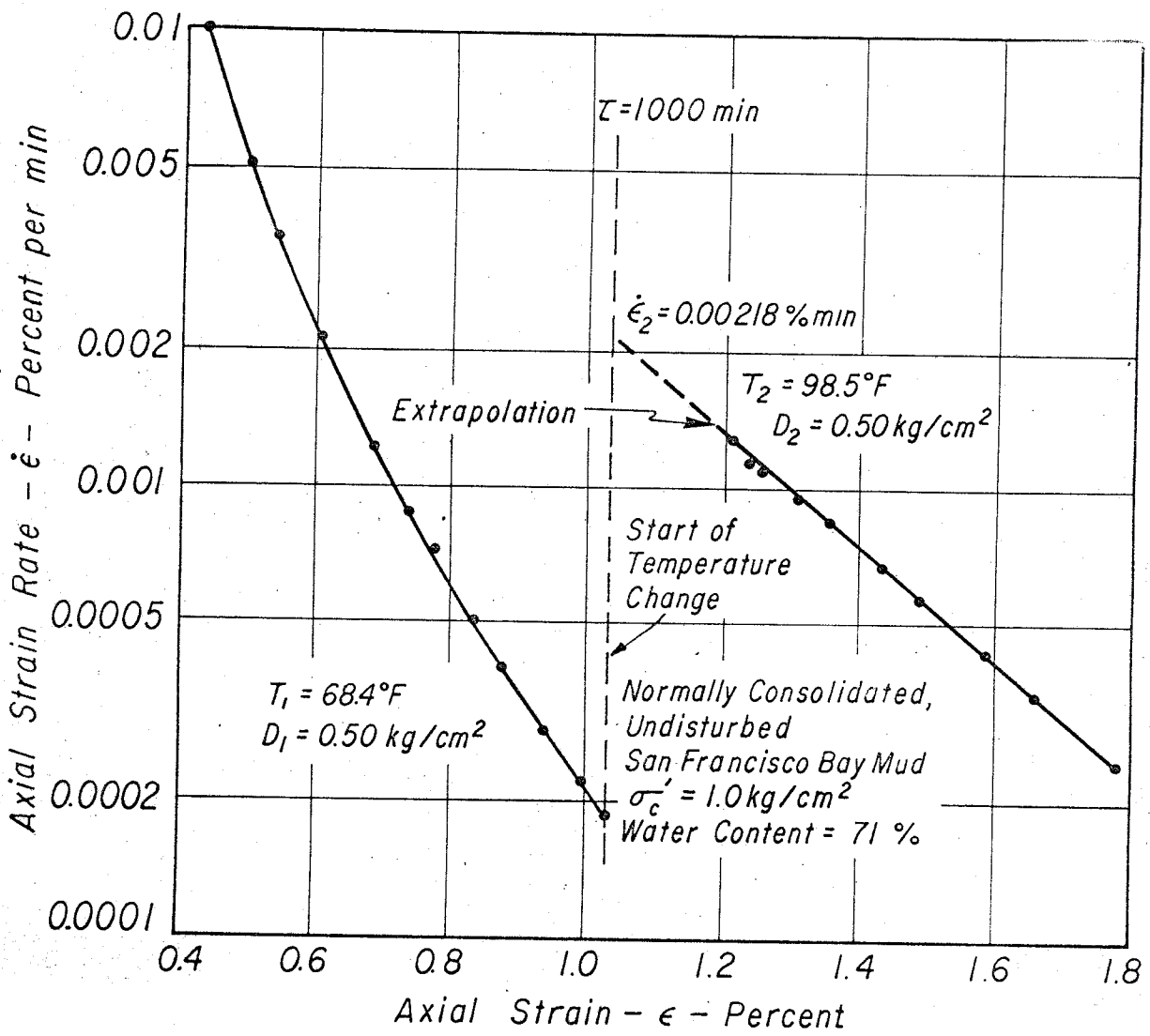


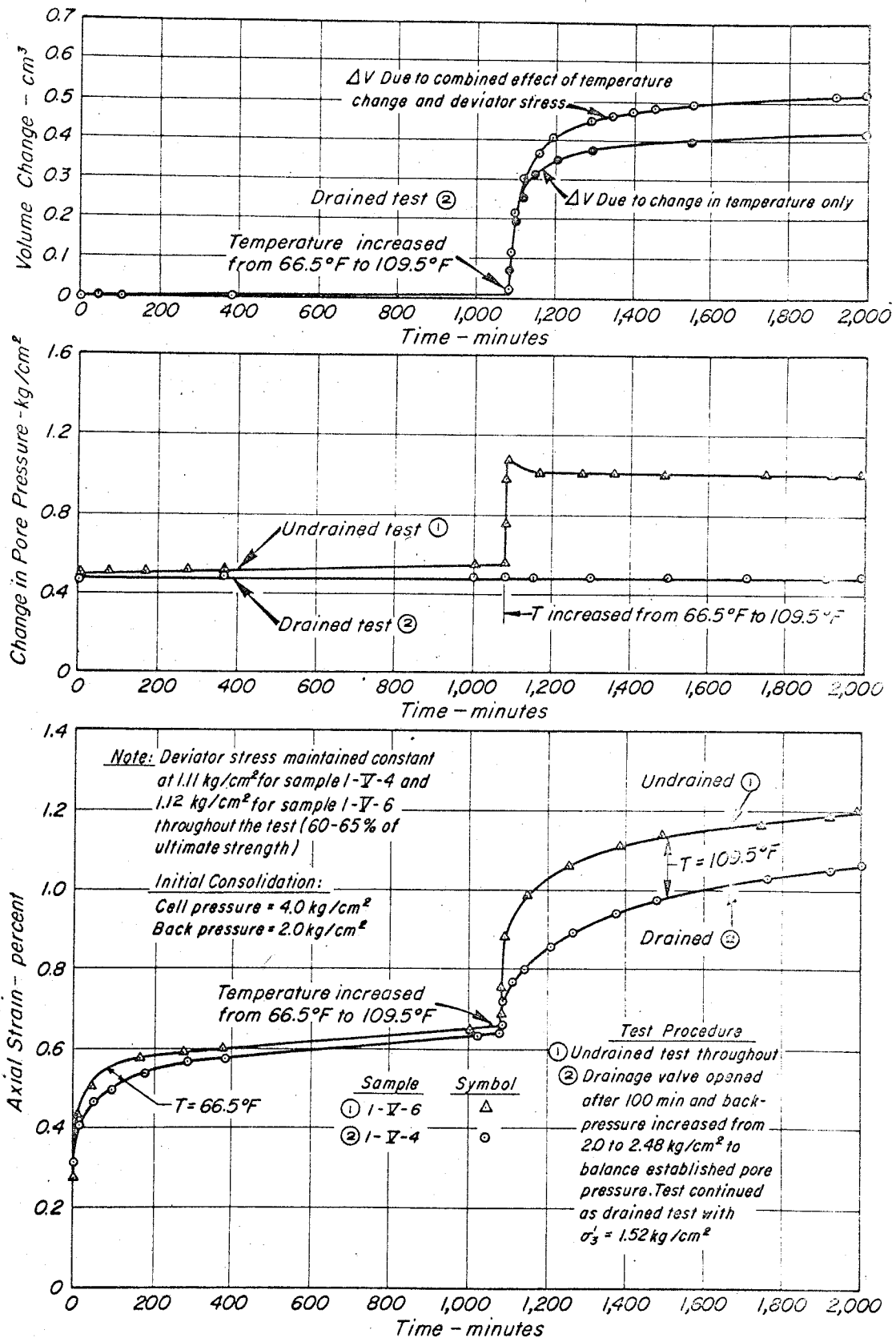
FIG. 5 RELATIONSHIP BETWEEN STRAIN RATE AND STRAIN FOR CREEP OF UNDISTURBED SAN FRANCISCO BAY MUD SUBJECTED TO RAPID TEMPERATURE CHANGE

Another aspect of importance in connection with tests of this type is that it is impossible to change the temperature of a clay without causing either a volume change or an effective stress change (Campanella and Mitchell (1967)).<sup>17</sup> The volume changes associated with changes of temperature of the magnitude used for this type of test (25°C or less) are relatively small, of the order of 1.0 percent or less of the initial sample volume. On the other hand the changes in effective stress caused by a temperature change under undrained conditions may amount to a factor of two or more. Since the effective stress in a clay can be expected to exert a major influence on the creep rate, meaningful determinations of the effect of temperature change alone on creep rates can only be obtained if the effective stresses are maintained constant.

Tests have shown, e.g., Mitchell and Campanella (1964),<sup>15</sup> that during undrained creep of saturated clay the application of the creep stress causes an immediate increase in pore water pressure but that after a relatively short time further changes in pore pressure with creep are very small. Thus for the creep tests reported herein the test was started under undrained conditions until the pore water pressure was essentially constant with time. The test was then converted to drained conditions by matching the pore pressure with a constant back pressure. This back pressure was maintained constant, as were also the cell pressure and deviator stress, throughout the temperature change and the remainder of the test. A typical result from this type of test is shown in Fig. 6. Data for an identical specimen

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<sup>17</sup>Campanella, R. G. and Mitchell, J. K. (1967) "Temperature Effects on Volume Changes and Pore Pressures in Soils," in press.



**Fig. 6 - THE EFFECT OF TEMPERATURE CHANGE ON THE CREEP BEHAVIOR OF SATURATED REMOLDED SPECIMENS OF ILLITE TESTED UNDER DRAINED AND UNDRAINED CONDITIONS.**

subjected to the same temperature variation but maintained undrained are also shown for comparison purposes.

It may be seen from Fig. 6 that when the temperature change was made under undrained conditions the pore water pressure approximately doubled. On the other hand for the drained test the temperature change resulted in a volume change of only about 0.5 cc, which was less than 0.6 percent of the sample volume of 90 cc. The considerably greater amount of creep observed for the undrained specimen after the temperature change would be expected because of the greatly reduced effective stress caused by the large pore water pressure increase. The factors influencing the magnitudes of pore pressure and volume change associated with temperature changes as discussed in detail by Campanella and Mitchell (1967).<sup>17</sup>

To determine if the variation of creep rate with temperature conformed with equation (10) identical specimens of San Francisco bay mud were subjected to a deviator stress of 0.45 kg per sq. cm. at a temperature,  $T_1$ , of 70°F. Creep was continued for a period of 200 minutes under these conditions and the temperature of each specimen was then increased by a different amount. Fig. 7 shows the variation of strain with time for these tests. At the end of creep at  $T_1$  the creep rates were as indicated by points a, b, and c in Fig. 8. The strain rates indicated by a', b', and c' are the values pertaining to the instant after the temperature increase. It may be seen that the data in Fig. 8 yield a good linear relationship between  $\log (\dot{\epsilon}/T)$  and  $\frac{1}{T}$  as predicted

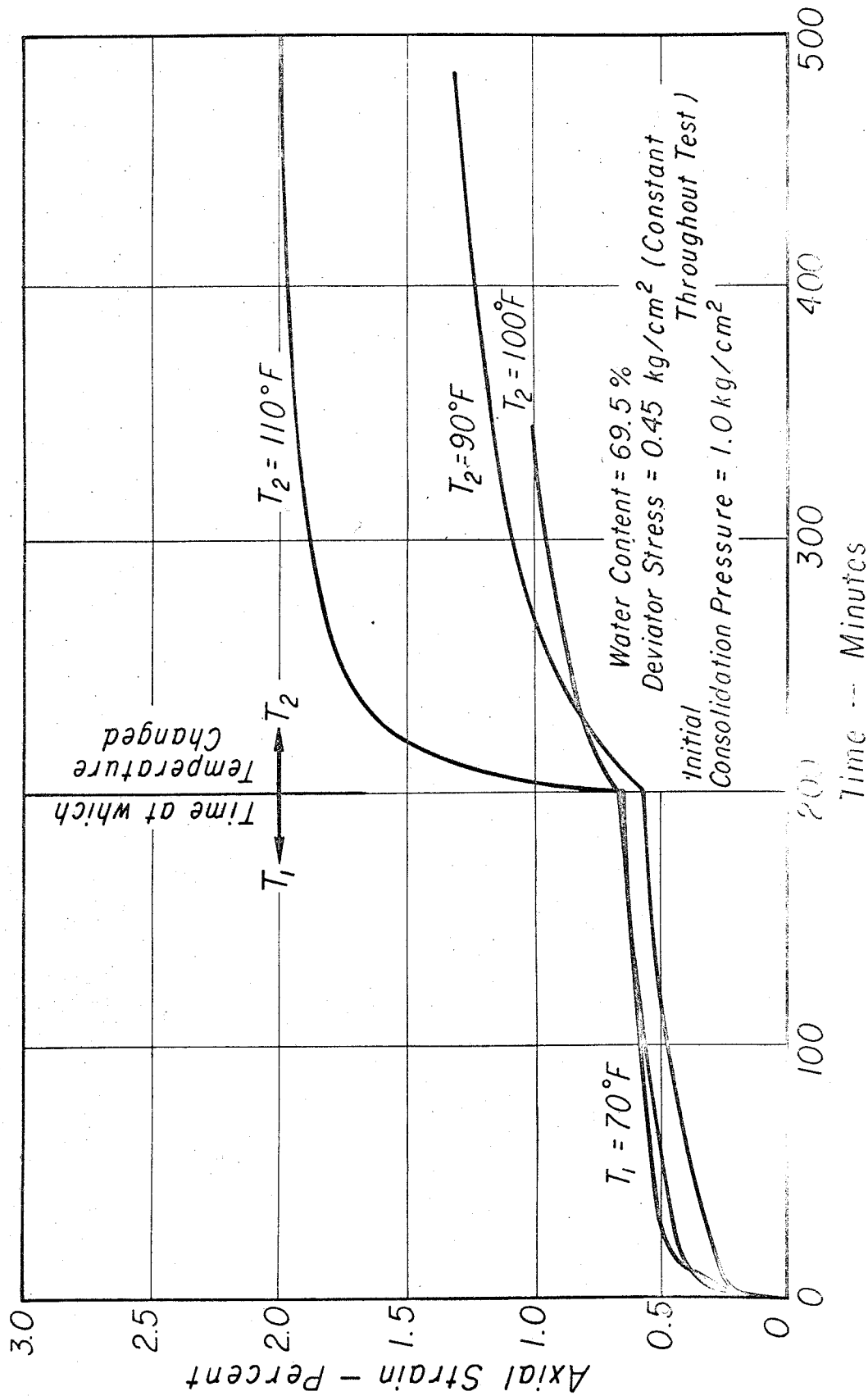


FIG. 7 EFFECT OF TEMPERATURE CHANGE ON THE CREEP OF  
 UNDISTURBED SAN FRANCISCO BAY MUD

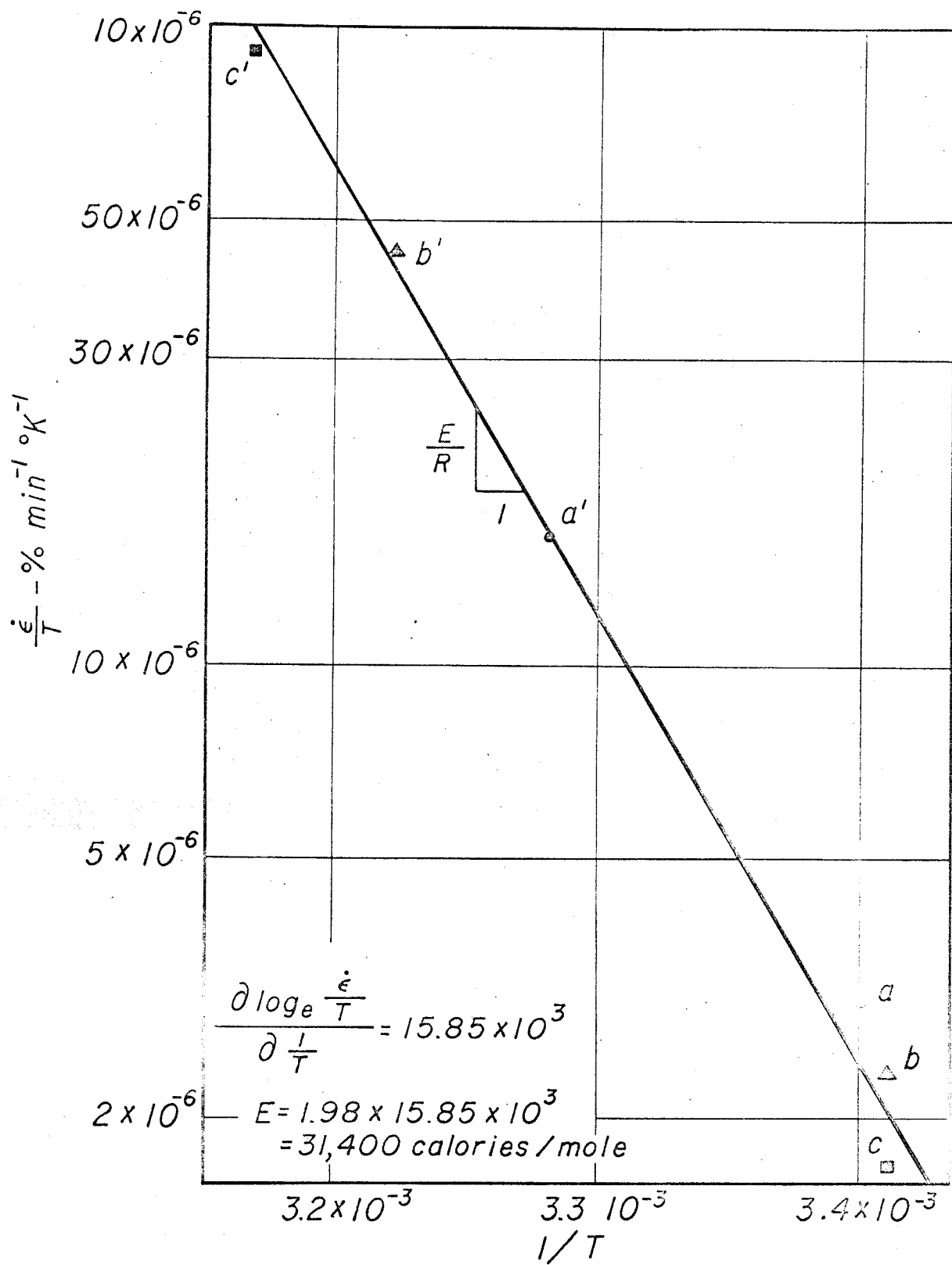


FIG. 8 STRAIN RATE AS A FUNCTION OF  $\frac{1}{T}$  FOR UNDISTURBED SAN FRANCISCO BAY MUD

by equation (10). From the slope of this line the experimental activation energy for creep may be determined to be 31,400 calories per mole.

These results lend support to the postulate that creep can be treated as a thermally activated process. It also appears that the temperature increment type of test on a single sample can be used for determination of the activation energy. For this type of test equation (8) can be used to give E directly according to

$$E = \frac{2.3 RT_2 T_1}{T_2 - T_1} \log_{10} \frac{\dot{\epsilon}_2 T_1}{\dot{\epsilon}_1 T_2} \quad (11)$$

#### THE EFFECT OF STRESS INTENSITY ON EXPERIMENTAL ACTIVATION ENERGY

Another test of the general applicability of equation (5) can be made by investigating the effect of deviator stress intensity on the experimental activation energy. For a given value of activation energy,  $\Delta F$ , the experimental activation energy should decrease linearly with increasing force per flow unit as indicated by equation (9). Whether or not  $\Delta F$  is itself stress or time independent cannot be stated with certainty. However, since it is a material property dependent on the bond strengths between flow units participating in the creep process, the assumption that it is essentially constant may not be unreasonable, at least for a given time after the start of creep.

The average force per flow unit should be directly proportional to the deviator stress provided variations in deviator stress do not cause large variations in structure. If this is the case then the experimental activation energy should decrease linearly with increasing

deviator stress. A series of saturated, remolded illite specimens were subjected to creep tests of the type described above at an initial temperature of 110°F. This temperature was selected because previous tests had shown for this material that the mean effective stress during creep would be nearly the same for all specimens over the entire range of creep deviator stresses. Thus any differences in behavior can be ascribed primarily to differences in deviator stress. All specimens were normally consolidated to 2.0 kg per sq. cm. prior to the creep tests. The mean effective stress after 1000 minutes of creep was  $1.42 \pm 0.03$  kg per sq. cm. Deviator stresses ranged from 0.35 to 1.58 kg per sq. cm., or about 20 to 90 percent of the initial compressive strength.

At the end of 1000 minutes of creep at a temperature of 110°F the temperature of each specimen was rapidly increased to 140°F and new creep rates determined. The value of experimental activation energy was then calculated for each specimen according to equation (11). Values of experimental activation energy as a function of deviator stress are shown in Fig. 9. It may be seen that all points except that for the lowest stress intensity define a straight line as predicted by equation (9).

Whether the point corresponding to a deviator stress of 0.35 kg per sq cm deviates from this line because of experimental error or a real difference in behavior is not known. It is known, however, that this stress intensity was sufficiently low as to invalidate the hyperbolic sine approximation used in equation (5), since the value of  $\left(\frac{f\lambda}{2kT}\right)$ , as determined from the variation of creep rate with stress intensity, was

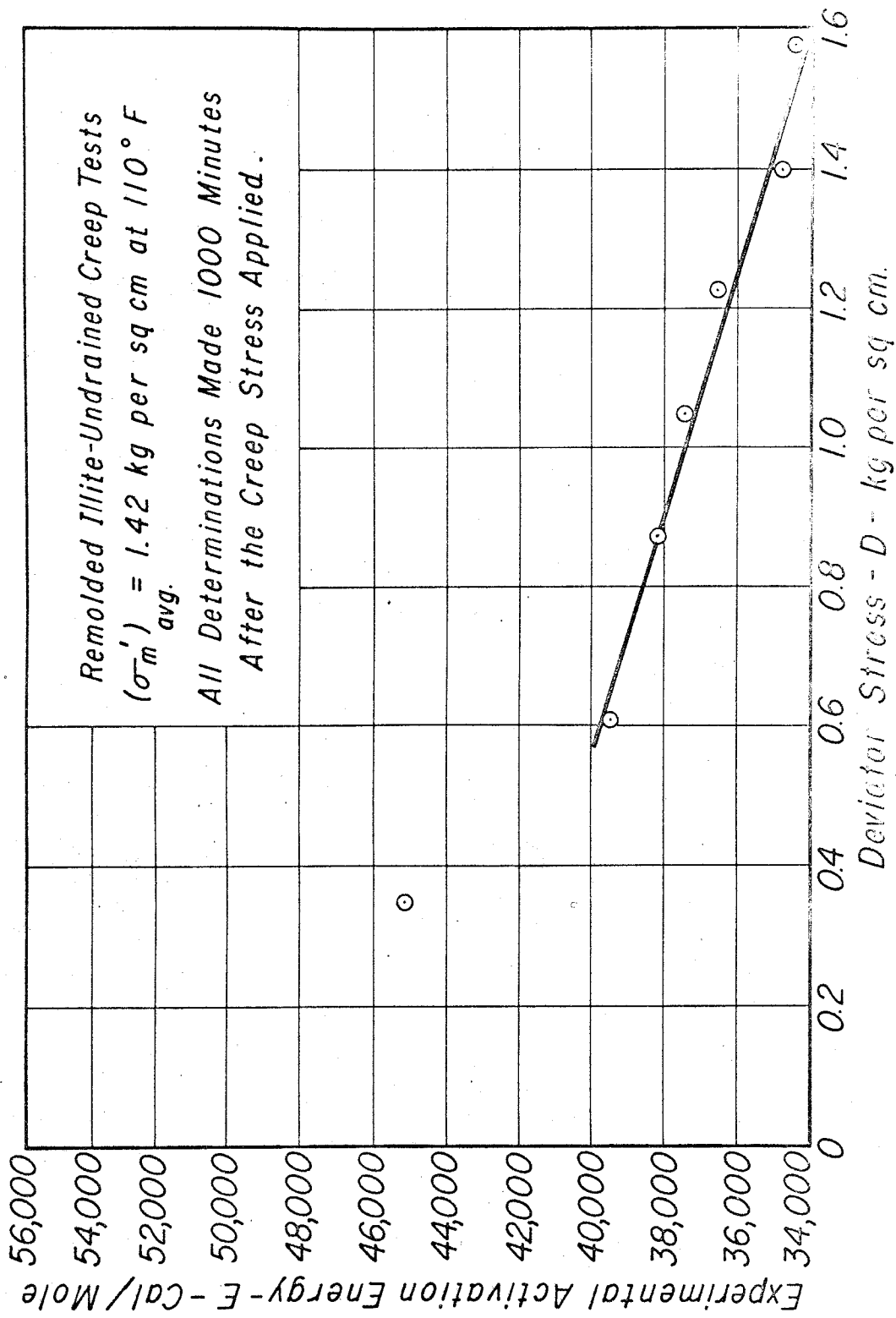


FIG.9 VARIATION IN EXPERIMENTAL ACTIVATION ENERGY WITH CREEP DEVIATOR STRESS

only about 0.85. Thus the use of equation (11) for determination of E for this particular test is not valid. It may be shown that use of the exact hyperbolic sine function would have led to a slightly smaller value for the experimental activation energy for this test, but would have virtually no effect on values corresponding to deviator stresses greater than about 0.5 kg per sq. cm.

Thus it appears reasonable to conclude that for stress intensities high enough to justify the use of equations (5) and (9) the observed and theoretical behavior agree very well from the standpoint of the variation of activation energy with creep stress.

#### THE INFLUENCE OF DEVIATOR STRESS ON CREEP RATE

An additional test of the basic rate equation is possible through investigation of the influence of stress intensity on creep rate. According to equation (3) if all other parameters are constant the strain rate should vary with the average stress,  $f$ , per flow unit according to

$$\dot{\epsilon} = K(t) \sinh \alpha' f \quad (12)$$

$$\text{where } K = 2X(t) \frac{kT}{h} \exp \left[ - \frac{\Delta F(t)}{RT} \right] \quad (13)$$

$$\text{and } \alpha' = \frac{\lambda}{2kT} \quad (14)$$

Equations (12) and (13) are written to include the possibility that both  $X$  and  $\Delta F$  may be time-dependent quantities. Although the basic mechanism of deformation may remain the same on an atomic scale, throughout the creep process, it is reasonable to expect that changes

in soil structure due to changes in particle orientations and inter-particle forces may cause variations in the activation energy,  $\Delta F$ , and the parameter  $X$ . The time after the start of creep appears to be the most useful single parameter for characterizing these variations as may be seen from the data presented below.

The form of equation (12) is shown in Fig. 10 by the curve labelled  $\sinh \alpha'f$ . The variation of  $\dot{\epsilon}/K$  to logarithmic scale with values of  $\frac{1}{2} \exp(\alpha'f)$  is also shown. It may be seen that for values of  $\alpha'f$  greater than about 1.5 the exponential and hyperbolic sine functions are almost identical. Thus the exponential approximation introduced into equation (5) causes virtually no error for values of  $\alpha'f$  greater than 1.5 and only small error for values as low as 1.0. The error increases rapidly, however, as the value drops below 1.0.

Fig. 11 shows the variation of the strain rate to logarithmic scale as a function of deviator stress intensity for undrained creep tests on remolded illite. For these tests identical specimens were consolidated isotropically under a stress of 2.0 kg per sq. cm. and then subjected to creep deviator stresses of different magnitudes. The data show for deviator stresses up to about 1.4 kg per sq. cm., or about 82 percent of the compressive strength at the start of the creep test, and for any given time after the start of creep, that the variation between creep rate and deviator stress agrees well in form with the prediction of equation (12). At values of deviator stress greater than 1.4 kg per sq. cm. the creep rate increases more rapidly with increase in stress than predicted by equation (12). Physical limitations on the strength of the material are responsible for this, since at

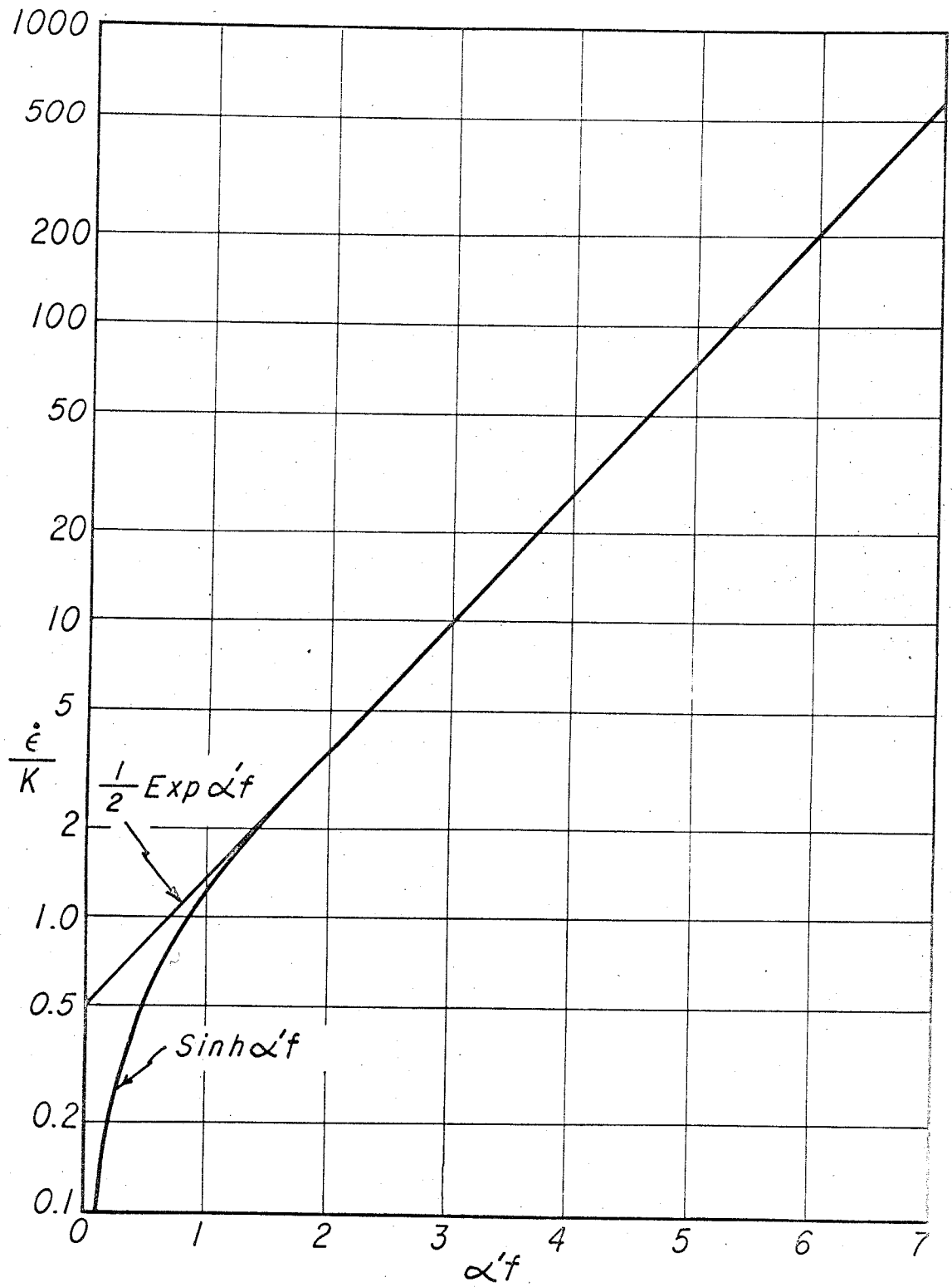


FIG.10  $\dot{\epsilon}/k$  vs  $\alpha'f$  ACCORDING TO HYPERBOLIC SINE AND EXPONENTIAL FUNCTIONS

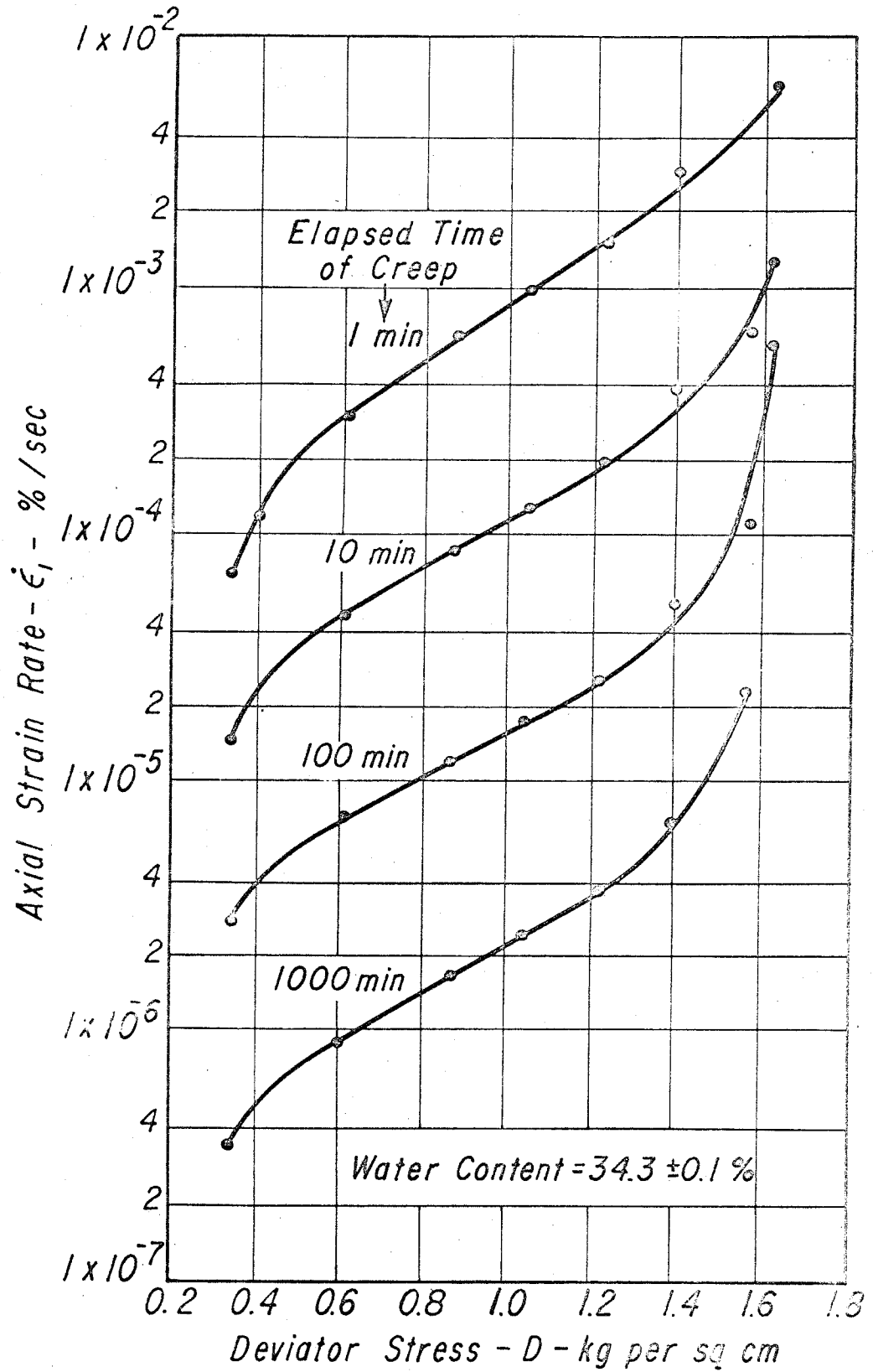


FIG. II VARIATION OF STRAIN RATE WITH DEVIATOR STRESS FOR UNDRAINED CREEP OF REMOLDED ILLITE.

these high stresses the clay is on the verge of failure and strain rates accelerate rapidly.

The data in Fig. 11 are shown in terms of the total deviator stress,  $D$ , applied to the specimen; whereas, equations (3), (5) and (12) are in terms of the average shear force per flow unit,  $f$ . If the number of flow units was the same for all specimens then  $f$  and  $D$  are related according to

$$f \propto \frac{D}{S}$$

where  $S$  represents the number of flow units. In terms of deviator stress equation (12) could be written

$$\dot{\epsilon} = K(t) \sinh \alpha D \quad (15)$$

where  $\alpha = \alpha'$ , and the constant of proportionality,  $a$ , takes into account the number of flow units and the necessary modification to convert deviator stress to a shearing stress.

For the range of stresses where the exponential approximation of strain rate is valid equation (15) can be written as

$$\dot{\epsilon} = \frac{K(t)}{2} \exp \alpha D \quad (16)$$

Thus

$$\ln \dot{\epsilon} = \ln \frac{K(t)}{2} + \alpha D \quad (17)$$

and the slope of the straight line sections can be used to evaluate  $\alpha$ , according to

$$\frac{\partial \ln \dot{\epsilon}}{\partial D} = \alpha \quad (18)$$

For the data shown in Fig. 11  $\alpha$  has the value of  $2.46 \text{ cm}^2$  per kg. Therefore, at a deviator stress of 0.5 kg per sq. cm.  $\alpha D$  amounts to 1.23, which as may be seen from Fig. 10, represents a value below which deviations from linearity between  $\ln \dot{\epsilon}$  and  $\alpha'f$  become noticeable.

Linear relationships between logarithm of strain rate and deviator stress for given times after the start of creep and for stresses greater than about 20 to 30 percent of the initial strength, but below those causing creep rupture, have been observed for a variety of clays. In addition to the data for saturated illite shown in Fig. 11 the authors have found similar relationships for dry illite, undisturbed normally consolidated (Fig. 2) and overconsolidated San Francisco bay mud, and for remolded San Francisco bay mud, all tested in undrained creep. The results of drained creep tests on undisturbed London clay (Bishop, 1966)<sup>18</sup> are shown in Fig. 12. Fig. 13 presents the results of creep tests on undisturbed Osaka alluvial clay (Murayama and Shibata, 1958).<sup>5</sup> Andersland and Akili<sup>13</sup> have also observed a linear relationship between logarithm of strain rate and creep stress for frozen soils at high stresses.

The fact that the relationship between logarithm of strain rate and deviator stress is linear and the slope is independent of time is in accordance with the dependencies predicted by equation (5). In all cases the least value of  $\alpha D$  is about 1.0 for the range of linearity between logarithm of strain rate and stress. This provides some degree of quantitative justification for the use of the exponential approximation in equation (5).

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<sup>18</sup>Bishop, A. W. (1966), "The Strength of Soils as Engineering Materials," Sixth Rankine Lecture, Geotechnique, Vol. XII, Number 2, June 1966.

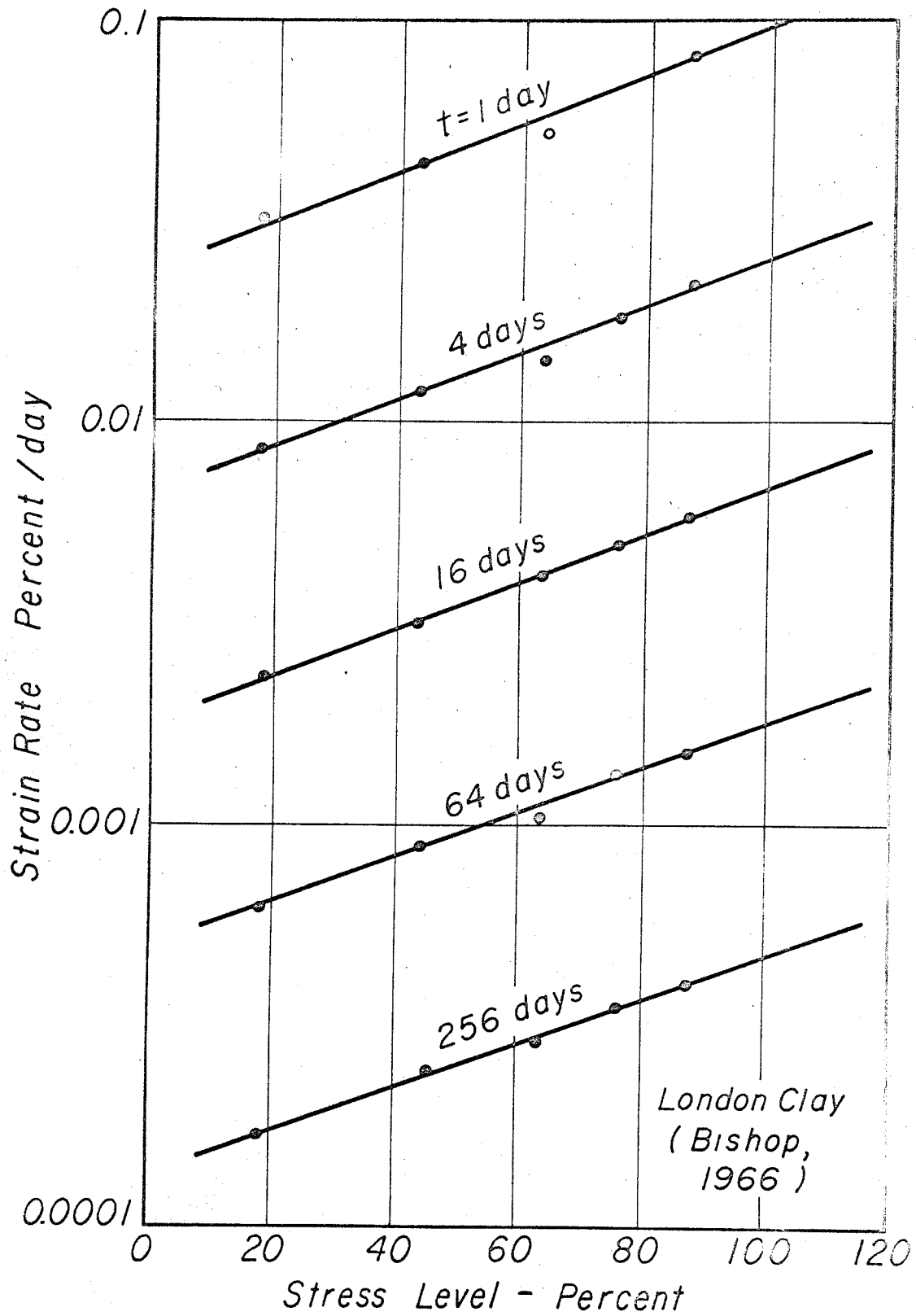


FIG.12 INFLUENCE OF STRESS LEVEL ON STRAIN RATE DURING DRAINED CREEP OF LONDON CLAY

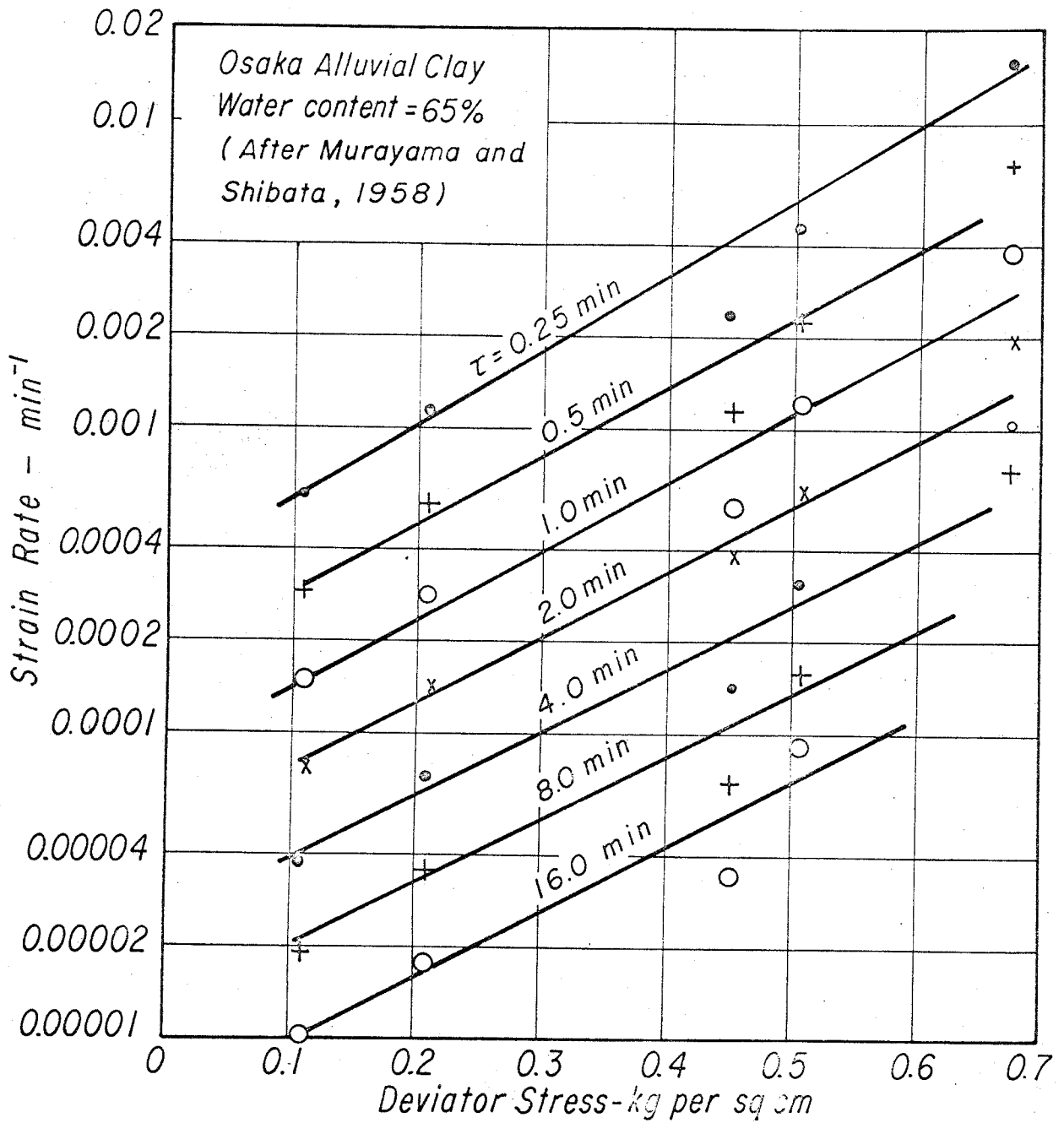


FIG. 13 INFLUENCE OF STRESS LEVEL ON STRAIN RATE DURING CREEP OF OSAKA ALLUVIAL CLAY

THE TIME DEPENDENCE OF CREEP RATE

The time-independent character of the dependence of creep rate on stress is clearly shown by Figs. 2, 11, 12 and 13. The data show, however, that at any given stress the creep rate decreases in a regular manner with time. The nature of this relationship is more clearly shown by Fig. 14, which gives the variation of strain rate with time for creep of illite, and by Fig. 1 for creep of undisturbed San Francisco bay mud. The regular nature of relationships of the type shown by Figs. 11 and 14 has been discussed in detail by Singh and Mitchell (1967),<sup>12</sup> and phenomenological equations have been proposed for the description of creep deformation over the range of stresses of engineering interest.

The basic form of equation (5) as regards the assumption of a thermally activated creep process, the relationship between experimental activation energy and deviator stress as stated by equation (9), and the predicted stress dependence of the creep rate would appear to be supported by the data presented above. It is clear however that equation (5) cannot properly describe creep behavior unless it is modified to account for the regular decrease in creep rate with time.

According to equation (13)

$$\frac{K(t)}{2} = X(t) \frac{kT}{h} \exp \left[ - \frac{\Delta F(t)}{RT} \right] \quad (19)$$

It may be deduced from plots of the type shown in Figs. 11 through 14 that  $\frac{K(t)}{2}$  may be given by (Singh and Mitchell, 1967)<sup>12</sup>

$$\frac{K(t)}{2} = A \left( \frac{t_1}{t} \right)^m \quad (20)$$

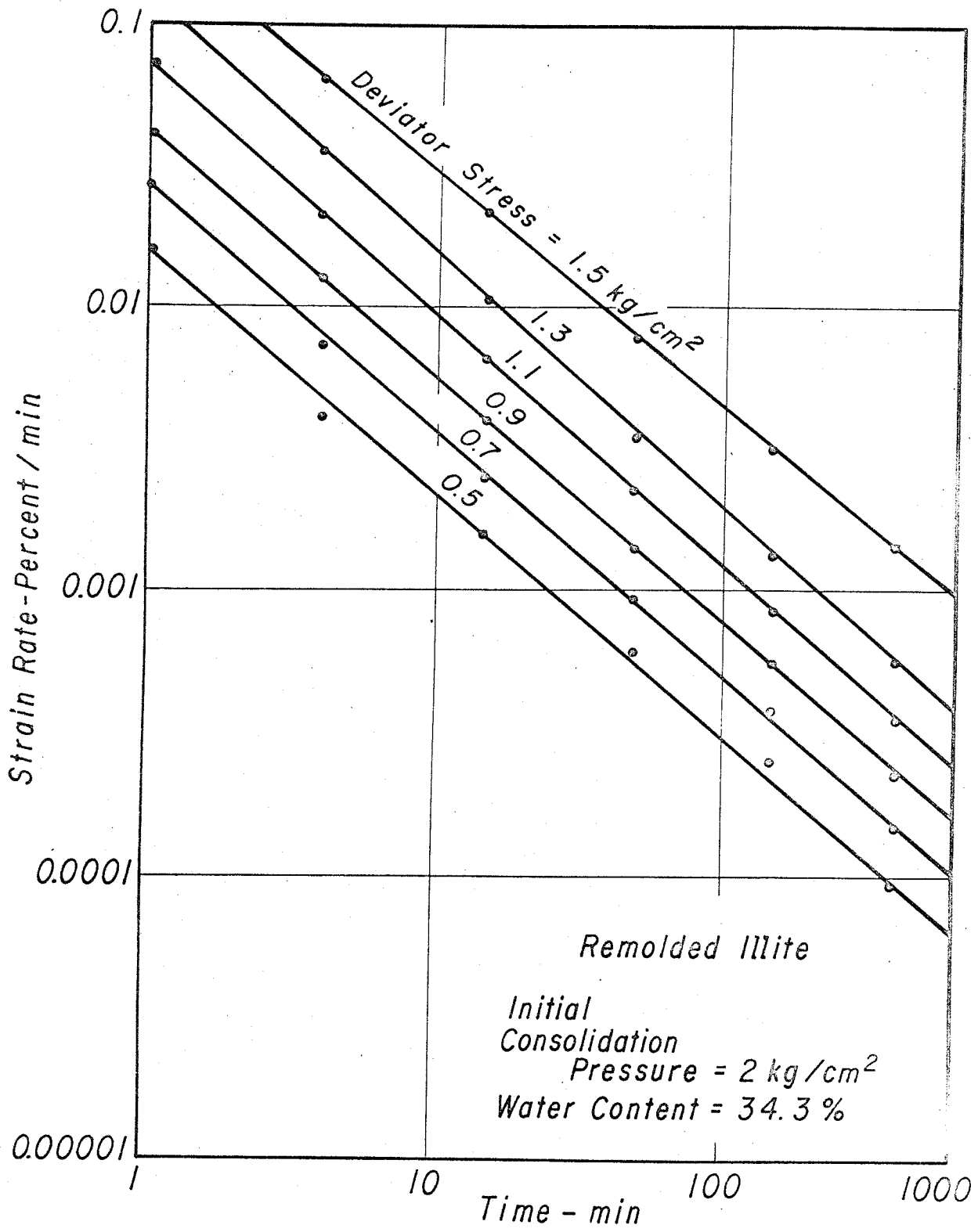


FIG.14 INFLUENCE OF TIME ON STRAIN RATE DURING CREEP OF REMOLDED ILLITE

where  $A$  is the strain rate at some arbitrarily chosen short time  $t_1$  and  $D = 0$ ; i.e., the value of strain rate corresponding to the projection of the  $\ln \dot{\epsilon}$  vs  $D$  line corresponding to time  $t_1$  to the value of  $D = 0$ , and  $m$  is the negative of the slope of the relationship between logarithm of strain rate and logarithm of time. It has been found for all data examined thus far that  $0.75 < m < 1.0$ .

Unfortunately it has not yet been possible to account completely for this time-dependence of the creep rate in quantitative physical terms relating to the rate process model for behavior. It is reasonable to conclude, however, that it is a function of changes in the microstructure which influence the parameters  $X$  and  $\Delta F$  as creep proceeds. Further discussion of the variation of structure during creep is given by Mitchell, Singh and Campanella (1967).<sup>19</sup> Thus for the study of general relationship between stress, creep strain rate and time and for the study of the parameter  $\alpha$  it would appear to be appropriate to use the following equation

$$\dot{\epsilon} = A \left( \frac{t_1}{t} \right)^m \exp (\alpha D) \quad (21)$$

For the evaluation of activation energy,  $\Delta F$ , the experimental activation energy,  $E$ , the parameter  $X$ , or the influence of temperature on behavior, it is necessary to compare specimens at the same time after the start of creep. In so doing, however, it must be borne in mind that quantities determined at any one time may not be the same at any other time.

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<sup>19</sup>Mitchell, J. K., Singh, A. and Campanella, R. G. (1967) "Bonding Effective Stress and Strength of Soils as Revealed by Creep Behavior," in press.

## CREEP BEHAVIOR AS AN INDICATOR OF DEFORMATION

## MECHANISMS IN SOILS

The fact that creep behavior is in general accord with the predictions of rate process theory, provided due account is taken of the time variations of some parameters, provides a basis for studying the fundamental character of deformation in soils. Determination of the activation energy for the creep process provides a method for investigating some of the characteristics of interparticle bonding.

The values of activation energy for creep in soils are of particular interest. The data presented herein indicate that the experimental activation energy,  $E$ , is a variable quantity which depends on the deviator stress. In addition  $E$  has been found to be a function of the elapsed time of creep (Mitchell, Singh and Campanella 1967).<sup>19</sup> It falls within a range of about 34,000 to 40,000 calories per mole for illite and is of the same order of magnitude for undisturbed San Francisco bay mud. It is reasonable to ask in connection with these values, "calories per mole of what?" The initial and final configuration of particles and atoms in the contacts between particles may involve many bonds. It may be necessary to do work in addition to simply breaking bonds if there is to be a displacement between two particles; e.g., it may be necessary to push other atoms, molecules, or particles out of the way. It might be argued that the necessity for simultaneous rupturing of several bonds would require an activation energy equal to the aggregate of the bond energies. Similarly, if the deformation required crossing a number of barriers in series then the activation

energy limiting the rate of movement would correspond to the highest barrier in the series, and it would be this value which would be determined by a measurement.

Ripple and Day (1966)<sup>20</sup> note, however, that most probably the barrier height is related to the breaking of a single bond. Low (1961)<sup>21</sup> points out, for example, that in the viscous flow of water the activation energy is approximately that for a single hydrogen bond even though each water molecule may form up to four hydrogen bonds with its neighbors depending on the temperature. Thus the values of activation energy can be interpreted to refer to the energy per mole of bonding sites.

If this interpretation is correct, then the shear force  $f$  which appears in the rate process equation must refer to the force per bond. A measure of the number of bonds participating in the creep process may be obtained from knowledge of the parameter  $\alpha$ , equation (17). This parameter may be determined by evaluating the stress dependence of the creep rate.

From equations (12), (14) and (15)

$$\alpha D = \alpha' f = \frac{\lambda}{2kT} f \quad (22)$$

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<sup>20</sup>Ripple, C. D. and Day, P. R. (1966) "Suction Responses Due to Homogeneous Shear of Dilute Montmorillonite - Water Pastes," Clay and Clay Minerals, Proceedings of the Fourteenth National Conference, Pergamon Press, pp. 307-316.

<sup>21</sup>Low, P. F. (1961) "Physical Chemistry of Clay-Water Interaction," Adv. Agron., Vol. 13, pp. 269-327.

and therefore

$$\alpha = \frac{\lambda}{2kT} \frac{f}{D} \quad (23)$$

But if there are  $S$  bonds per unit area on which the shear stress acts, and it is noted that the maximum shear stress in a triaxial specimen is equal to  $D/2$  then as a first approximation  $f$  and  $D$  can be assumed related to each other according to

$$f = \frac{D}{2S} \quad (24)$$

On this basis equation (23) becomes

$$\alpha = \frac{\lambda}{4SkT} \quad (25)$$

and knowledge of  $\alpha$  provides a basis for obtaining a measure of the number of bonds per unit area provided a reasonable assumption can be made for  $\lambda$ , the distance between bonding sites, Fig. 3. This approach has been used for the study of the effects of such factors as consolidation pressure, overconsolidation, and disturbance on the mechanism of deformation of soils. This study has led to the formulation of a general hypothesis for the relationships between inter-particle bonding, effective stress and strength of soils which is presented in a companion paper (Mitchell, Singh, and Campanella, 1967).<sup>19</sup>

The relatively high value of activation energy for creep in clays is considered significant. Primary valence bonds; e.g. ionic, covalent, are generally observed to have activation energies greater than 10,000 cal. per mole, and the activation energies of chemical reactions are generally in the range 10-100 k cal. per mole. Activation energies for flow of several materials have been reported as follows:

<u>Material</u>	<u>Activation Energy Kcal/Mole</u>	<u>Reference</u>
Water	4-5	Glasstone, Laidler and Eyring (1941) <sup>4</sup>
Plastics	7-14	Ree and Eyring (1958) <sup>22</sup>
Asphalt	14-20	Herrin and Jones (1963) <sup>23</sup>
Soils	25-45	Authors, Ripple and Day (1966) <sup>20</sup>
Concrete	54	Polivka and Best (1960) <sup>24</sup>
Metals	50+	Finnie and Heller (1959) <sup>25</sup>
Frozen Soils	93.6	Andersland and Akili (1967) <sup>13</sup>

Unfortunately it is not always clearly indicated whether the reported values are free energies of activation,  $\Delta F$ , or experimental values,  $E$ , which are less than  $\Delta F$  by the amount of energy contributed by the deforming stress, equation (9). Activation energy values for clay deformation have also been reported by Murayama and Shibata (1958, 1961, 1966)<sup>5,6,7</sup> and Christensen and Wu (1964).<sup>9</sup> The values reported are in the same general range as listed above. They were determined through the use of a rheological model for description of the deformation process, which introduces some artificiality into their analyses because of the mathematical constraints imposed. The procedure used

<sup>22</sup>Ree, R. and Eyring, H. (1958) "The Relaxation Theory of Transport Phenomena," Chap. 3 of Rheology, Vol. II, F. R. Eyring, Editor, Academic Press, N. Y.

<sup>23</sup>Herrin, M. and Jones, G. (1963), "Behaviour of Bituminous Materials from the Viewpoint of Absolute Rate Theory," Amer. Assoc. of Asphalt Paving Technologists, Feb. 1963.

<sup>24</sup>Polivka, M. and Best, C. (1960), "Investigation of the Problems of Creep in Concrete by Dorn's Method," Univ. of Calif., Berkeley, California, 1960.

<sup>25</sup>Finnie, I. and Heller, W. (1959) Creep of Engineering Materials, McGraw-Hill, 1959.

herein is free of any assumptions concerning the elastic characteristics of the soil structure or distribution of applied stress between elastic and flowing elements.

, The fact that the activation energy for creep of soils is from five to ten times that for viscous flow of water suggests that interparticle slippage through water films at interparticle contact points is not likely -- at least not through any sort of normal water films. Furthermore creep tests on dried illite specimens have given essentially the same values of activation energy as tests on saturated illite. Now if the activation energy values refer to the energy per mole of bonding sites each bond must have an energy of magnitude in the primary valence chemical bonding range. This must mean in turn that interparticle contacts in soils are essentially solid to solid. Such a contact could be actual mineral to mineral or mineral through solid adsorbed layer to mineral.

Bonds at interparticle contacts could be expected to form as the result of transfer of compressive stress at the contact. This stress may be derived from compressive stresses applied to the soil structure or from interparticle attractions. If the number of bonds formed is directly dependent on the compressive force transferred at a contact and each bond on the average offers the same resistance to shear, then the development of shearing resistance between soil grains could be considered analogous to the mobilization of friction between solid bodies as given by the Bowden-Tabor (1954)<sup>26</sup> theory. The shearing resistance of a soil mass on a macroscopic scale, however, must involve

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<sup>26</sup>Bowden, F. P. and Tabor, D. (1954) The Friction and Lubrication of Solids, Oxford, Clarendon Press, 392 pp.

other factors such as dilatency and particle bending and crushing.

#### CONCLUSIONS

The fundamental equation of the theory of absolute reaction rates has been developed and adapted for use in the study of time-dependent deformation of soils. This theory, known also as rate process theory, has been used previously by several investigators. Heretofore little direct evidence has been presented which supports the application of this theory to soils, since earlier studies have assumed it to be valid a priori. From the results of carefully controlled triaxial creep tests on undisturbed and remolded clays over a range of temperature and stress conditions it has been possible to establish the following facts.

1. The temperature dependence of creep rate is in conformity with the theoretical predictions, since a plot of the logarithm of the strain rate divided by the temperature versus the reciprocal of the absolute temperature is linear and has a negative slope. The experimental activation energy is directly proportional to this slope. Thus creep of soils can be treated as a thermally activated process.
2. The experimental activation energy decreases linearly with increase in creep shear stress, in accordance with the theory.
3. For sub-failure shear stresses the variation of creep rate with creep stress follows the hyperbolic sine function given by theory.

4. At stresses greater than about 20 to 30 percent of the shear strength the logarithm of strain rate after any given period of creep is directly proportional to the creep stress. The constant of proportionality is independent of time. The hyperbolic sine function degenerates to a simple exponential function for values of the argument greater than about 1.5 and is a good approximation in the range 1.0 to 1.5. In all cases where the linear relationship between logarithm of strain rate and stress was observed the value of the slope times the stress was 1.0 or greater; i.e., the exponential approximation of the hyperbolic sine was justified.
5. The creep rate is time dependent, decreasing according to an inverse power function of time as given by equation (21). Thus, while this study has supported the validity of the temperature, activation energy, and stress terms of the rate process equation (equation 5), further study is needed for proper interpretation of the physical factors controlling the variation of the parameters  $X$  and  $\Delta F$ , eq. (5), with time. Some further analysis of these quantities is given by Mitchell, Singh and Campanella (1967).<sup>19</sup>
6. Under conditions of constant temperature the creep rate can be expressed by the simple three parameter equation (21); i.e.,

$$\dot{\epsilon} = A \left( \frac{t_1}{t} \right)^m \exp (\alpha D) ,$$

at least over the range of stress of usual engineering interest. It would appear desirable that any spring and dashpot rheological

model proposed for the description of soil creep should be capable of giving a response compatible with this relationship. The authors have not yet been able to devise such a model.

7. The study of creep behavior of soils provides a means for examination of mechanisms of deformation in soils, primarily through evaluation of the activation energy and the number of bonds per unit area under different conditions of consolidation, water, overconsolidation and disturbance. (Mitchell, Singh and Campanella, 1967).<sup>19</sup>
8. Consideration of the activation energy for creep of clays suggests that interparticle bonding is probably of the primary valence type, and the resistance to shear cannot be developed by viscous water films; i.e., interparticle contacts must be effectively solid to solid.

#### ACKNOWLEDGEMENTS

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